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Homotopy genus of BU and the Bott map

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1. Introduction

The homotopy genus of a nilpotent finite CW-complex X is defined as follows ([5], [7]):

$$\{\,[Y]\,|\,Y \simeq_p X \text{ for each prime } p\,\}.$$

The homotopy genus of certain spaces are computed, for example, the order of the homotopy genus of a classifying space of a compact connected Lie group is uncountable infinite. But the homotopy genus of $BU = BU(\infty)$ is not known yet. The purpose of this paper is to determine the homotopy genus of the pair of BU and the Bott map of BU. The main theorem below says that it is unique.

Theorem. *Let X be a pointed of finite type simply connected CW-complex equipped with a map $\lambda : S^2 \wedge X \to X$ and a homotopy equivalences $h_p : X_{(p)} \to BU_{(p)}$ for each prime p such that they satisfy the following homotopy commutative diagram*

$$\begin{tikzcd}
(S^2 \wedge X)_{(p)} \arrow[r, "1\wedge h_p"] \arrow[d, "\lambda_{(p)}"'] & (S^2 \wedge BU)_{(p)} \arrow[d, "\beta_{(p)}"] \\
X_{(p)} \arrow[r, "h_p"'] & BU_{(p)},
\end{tikzcd}$$

where $\beta : S^2 \wedge BU \to BU$ is the Bott map. Then we have a homotopy equivalence $h : X \xrightarrow{\sim} BU$ which satisfies the following homotopy commutative diagram.

$$\begin{tikzcd}
S^2 \wedge X \arrow[r, "1\wedge h"] \arrow[d, "\lambda"'] & S^2 \wedge BU \arrow[d, "\beta"] \\
X \arrow[r, "h"'] & BU
\end{tikzcd}$$

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