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KO-theory of complex Stiefel manifolds

By

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1. Introduction

The purpose of this paper is to determine the KO^*-group of the complex Stiefel manifold $V_{n,q}$ which is q-frames in $\mathbf{C}^n$. We compute it by making use of the Atiyah-Hirzebruch spectral sequence of $KO^*(V_{n,q})$ and obtain the following:

Theorem. *Let a_k, b_k and c_k be as follows, then we have*

$$KO^{-i}(V_{n,q}) \cong a_k\mathbf{Z} \oplus c_k\mathbf{Z}_2$$

for $n = 2k$, $q \leq 4$ and $n = 2k+1$, $q \leq 3$, and

$$KO^{-i}(V_{n,q}) \cong KO^{-i-4}(V_{n,q}) \cong a_k\mathbf{Z} \oplus b_k\mathbf{Z}_2$$

for otherwise.

q	a_0	a_{-1}	a_{-2}	a_{-3}
$2l$	$2^{2l-2}+2^{l-1}$	2^{2l-2}	$2^{2l-2}-2^{l-1}$	2^{2l-2}
$2l+1$	$2^{2l-2}+2^{l-1}$	$2^{2l-2}-(-1)^n 2^{l-1}$	$2^{2l-2}-2^{l-1}$	$2^{2l-2}+(-1)^n 2^{l-1}$

(n,q)	b_0	b_{-1}	b_{-2}	b_{-3}
$(2k,2l)$	2^{q-4}	$3\cdot 2^{q-4}$	$3\cdot 2^{q-4}$	2^{q-4}
$(2k,2l+1)$	0	2^{q-3}	2^{q-2}	2^{q-3}
$(2k+1,2l)$	0	2^{q-2}	2^{q-2}	0
$(2k+1,2l+1)$	2^{q-3}	2^{q-2}	2^{q-3}	0

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