J. Math. Kyoto Univ. (JMKYAZ)
48-1 (2008), 7–26

Subsheaves of a hermitian torsion free coherent sheaf on an arithmetic variety

By

Atsushi Moriwaki

Introduction

Let K be a number field and O_K the ring of integers of K. Let (E, h) be a hermitian finitely generated flat O_K-module. For an O_K-submodule F of E, let us denote by $h_{F \hookrightarrow E}$ the submetric of F induced by h. It is well known that the set of all saturated O_K-submodules F with $\widehat{\deg}(F, h_{F \hookrightarrow E}) \geq c$ is finite for any real numbers c (for details, see [4, the proof of Proposition 3.5]).

In this note, we would like to give its generalization on a projective arithmetic variety. Let X be a normal and projective arithmetic variety. Here we assume that X is an arithmetic surface to avoid several complicated technical definitions on a higher dimensional arithmetic variety. Let us fix a nef and big C^∞-hermitian invertible sheaf $\overline{H}$ on X as a polarization of X. Then we have the following finiteness of saturated subsheaves with bounded arithmetic degree, which is also a generalization of a partial result [5, Corollary 2.2].

Theorem A (cf. Theorem 3.1). *Let E be a torsion free coherent sheaf on X and h a C^∞-hermitian metric of E on $X(\mathbb{C})$. For any real number c, the set of all saturated $\mathcal{O}_X$-subsheaves F of E with $\widehat{\deg}(\widehat{c}_1(\overline{H}) \cdot \widehat{c}_1(F, h_{F \hookrightarrow E})) \geq c$ is finite.*

For a non-zero C^∞-hermitian torsion free coherent sheaf $\overline{G}$ on X, the *arithmetic slope* $\hat{\mu}_{\overline{H}}(\overline{G})$ of $\overline{G}$ with respect to $\overline{H}$ is defined by

$$\hat{\mu}_{\overline{H}}(\overline{G}) = \frac{\widehat{\deg}(\widehat{c}_1(\overline{H}) \cdot \widehat{c}_1(\overline{G}))}{\operatorname{rk} G}.$$

As defined in the paper [5], (E, h) is said to be *arithmetically μ-semistable* with respect to $\overline{H}$ if, for any non-zero saturated $\mathcal{O}_X$-subsheaf F of E,

$$\hat{\mu}_{\overline{H}}(F, h_{F \hookrightarrow E}) \leq \hat{\mu}_{\overline{H}}(E, h).$$

The above semistability yields an arithmetic analogue of the Harder-Narasimham filtration of a torsion free sheaf on an algebraic variety as follows: A filtration

$$0 = E_0 \subsetneq E_1 \subsetneq \cdots \subsetneq E_l = E$$

Received December 11, 2006