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ON THE DIMENSION OF THE STABILITY GROUP FOR A LEVI NON-DEGENERATE HYPERSURFACE

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ABSTRACT. We classify locally defined Levi non-degenerate non-spherical real-analytic hypersurfaces in complex space for which the dimension of the group of local CR-automorphisms has the second largest positive value.

1. Introduction

Let M be a real-analytic hypersurface in \mathbb{C}^{n+1} passing through the origin. Assume that the Levi form of M at 0 is non-degenerate and has signature (n-m,m) with $n \geq 2m$. Then in some local holomorphic coordinates $z = (z_1, \ldots, z_n), w = u + iv$ in a neighborhood of the origin, M can be written in the Chern-Moser normal form (see [CM]), that is, given by an equation

$$v = \langle z, z \rangle + \sum_{k, \overline{l} \geq 2} F_{k\overline{l}}(z, \overline{z}, u),$$

where $\langle z, z \rangle = \sum_{\alpha,\beta=1}^{n} h_{\alpha\beta} z_{\alpha} \overline{z_{\beta}}$ is a non-degenerate Hermitian form with signature (n-m,m), and $F_{k\overline{l}}(z,\overline{z},u)$ are polynomials of degree k in z and \overline{l} in \overline{z} whose coefficients are analytic functions of u such that the following conditions hold:

(1.1)
$$\begin{array}{rrrr} \operatorname{tr} F_{2\overline{2}} &\equiv 0, \\ \operatorname{tr}^2 F_{2\overline{3}} &\equiv 0, \\ \operatorname{tr}^3 F_{3\overline{3}} &\equiv 0. \end{array}$$

Here the operator tr is defined as

$$\operatorname{tr} := \sum_{\alpha,\beta=1}^{n} \hat{h}_{\alpha\beta} \frac{\partial^2}{\partial z_{\alpha} \partial \overline{z}_{\beta}},$$

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