ILLINOIS JOURNAL OF MATHEMATICS
Volume 31, Number 3, Fall 1987

# THE HEREDITARY DUNFORD-PETTIS PROPERTY ON $C(K, E)$

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A Banach space $E$ is said to have the Dunford-Pettis property if for every pair of weakly null sequences $(x_n) \subset E$ and $(x'_n) \subset E'$ one has $\lim\langle x_n, x'_n \rangle = 0$. Following Diestel [1] we shall say that a Banach space $E$ is hereditarily Dunford-Pettis (or also that $E$ has the hereditary Dunford-Pettis property) if all of its closed subspaces have the Dunford-Pettis property. The first known example of a space enjoying this property was $c_0$ [3]. Besides $c_0$, the most simple examples of these spaces are $c_0(\Gamma)$ for any set $\Gamma$ and Schur spaces. Practically the rest of the known examples are among the $C(K)$ spaces (see Theorem 1).

In this paper we characterize when $C(K, E)$, the Banach space of all continuous functions defined on a compact Hausdorff space $K$ with values in a Banach space $E$, endowed with the supremum norm, has the hereditary Dunford-Pettis property.

The notations and terminology used and not explained here can be found in [1], [5], [7].

Recall that if $K$ is a compact Hausdorff space the $\omega$-th derived set of $K$ is defined by

$$K^{(\omega)} = \bigcap_{n=1}^{\infty} K^{(n)},$$

where $K^{(0)} = K$ and $K^{(n)}$ is the set of all accumulation points of $K^{(n-1)}$ for $n \in \mathbf{N}$; and $K$ is said to be dispersed or scattered if it does not contain any perfect set.

The following characterization of hereditarily Dunford-Pettis $C(K)$ spaces is due essentially to Pelczynski and Szlenk (see [1], [6]).

THEOREM 1. *Let $K$ be a compact Hausdorff space. Then $C(K)$ has the hereditary Dunford-Pettis property if and only if $K$ is dispersed and the $\omega$-th derived set of $K$ is empty.*

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Received May 20, 1985.
[1]Supported in part by CAICYT grant 0338-84 (Spain).