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CODIMENSION 2 SUBSCHEMES OF PROJECTIVE SPACES WITH STABLE RESTRICTED TANGENT BUNDLE AND LIAISON CLASSES

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Introduction

Let $C \subset \mathbf{P}^n$ be a smooth complex curve. Several papers have considered the problem of the stability of the restricted tangent bundle $T\mathbf{P}^n \mid C$ (e.g., see [HK] and the references quoted there). Using several results and methods from linkage theory we first show how to construct, in any even linkage class, many curves in $\mathbf{P}^3$ such that $T\mathbf{P}^3 \mid C$ is stable. Then a corresponding result is given for even linkage classes of codimension 2 locally Cohen-Macaulay subschemes of $\mathbf{P}^n$. For the required background and main results on linkage theory a very good reading is found in [MP1] for the case $n = 3$ and [Mi] for the general case.

Recall that any even linkage class, $\boldsymbol{L}$, can be decomposed as a disjoint union $\boldsymbol{L} = \boldsymbol{L}_0 \cup \boldsymbol{L}_1 \cup \boldsymbol{L}_2 \cup \cdots$ based on the shift of the various elements of $\boldsymbol{L}$ (see [Mi, Def. 4.3.5]).

Here is our main result.

THEOREM 0.1. *Assume $\boldsymbol{P}^n = \boldsymbol{P}^n_{\boldsymbol{K}}$ where $\boldsymbol{K}$ is an algebraically closed field of characteristic* 0. *Let $\boldsymbol{L} = \boldsymbol{L}_0 \cup \boldsymbol{L}_1 \cup \boldsymbol{L}_2 \cup \cdots$ be an even linkage class of equidimensional codimension* 2 *locally Cohen-Macaulay subschemes of $\boldsymbol{P}^n$ (decomposed in the usual way with respect to shift). Then there is an integer x such that for all integers $t \geq x$ there is an integral locally Cohen-Macaulay subscheme $X_t \in \boldsymbol{L}_t$ with* $\dim(\mathrm{Sing}(X)) \leq n - 6$ *and with $T\boldsymbol{P}^n \mid X_t$ stable.*

In particular, if $n \leq 5$ then any X_t as in the statement of Theorem 0.1 is smooth. First we will prove Theorem 0.1 in the case $n = 3$. The general case will be similar, except for the terminology and except at one point in which we will reduce to the case $n = 3$. A nice feature of the theory of linkage is that we know several interesting examples, in particular in the case of space curves. We will point out these examples in a few remarks.

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