THE σ-SYMBOL OF THE SINGULAR INTEGRAL OPERATORS OF CALDERÓN AND ZYGMUND

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In this paper we extend the σ-symbol of [2]. Our extension is a homomorphism of a C^*-subalgebra, $\mathfrak{A}$, of bounded operators on $L^2(R^n)$ onto the bounded continuous functions on $R^n \times S^{n-1}$. The kernel of this homomorphism is the set of all T such that ψT and $T\psi$ are compact operators for each $\psi \in C_0^\infty(R^n)$. We also show that $\mathfrak{A}$ and σ are uniquely determined by these properties.

Cordes [3] and Seeley [7] have considered σ on a smaller algebra than $\mathfrak{A}$ and obtained a homomorphism onto the continuous functions on $S^n \times S^{n-1}$, whose kernel is the compact operators. Our results, which extend theirs, were obtained after reading their papers.

Our results are stated precisely in §1. The information used about singular integral operators is discussed in §2. All of it is contained in [2]. The seminar notes [1] also contain this information.

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1. The statement of the main results

R^n will always denote Euclidean n-space ($n > 1$) and S^{n-1} will be the unit sphere in R^n. We use $\langle\ ,\ \rangle$ for the usual scalar product in R^n and $\|\quad\|$ for the corresponding norm. The word function will always mean a complex-valued function. We denote the coordinate functions on R^n by $u_1, \cdots, u_n$ and if $\alpha = (\alpha_1, \cdots, \alpha_n)$ where the α_j are nonnegative integers, we write

$$u_\alpha = u_1^{\alpha_1} \cdots u_n^{\alpha_n} \quad \text{and} \quad D_\alpha = \frac{\partial^{\alpha_1+\cdots+\alpha_n}}{\partial u_1^{\alpha_1} \cdots \partial u_n^{\alpha_n}}.$$

We use the standard notation $C^\infty(R^n)$ for the set of functions defined on R^n, whose partial derivatives of all orders exist and are continuous, and use $C_0^\infty = C_0^\infty(R^n)$ for functions in $C^\infty(R^n)$ that have compact support.

We also use the notation $BC = BC(R^n \times S^{n-1})$ for the set of all bounded continuous functions on $R^n \times (R^n - [0])$ such that $k(x, \lambda y) = k(x, y)$ for all $\lambda > 0$. Thus BC is essentially the same as the bounded continuous functions on $R^n \times S^{n-1}$. If U is any open set in R^n, we also write

$$B^\infty(U) = [f \in C^\infty(U) : D_\alpha f \text{ is bounded for every } \alpha].$$

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