PERIODIC TRANSFORMATIONS OF 3-MANIFOLDS[1]

BY

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1. M will denote a triangulated 3-manifold, G a finite group, (G, M) an effective simplicial action, *orientation-preserving whenever M is orientable.* Concerning the action we assume that (1) for every $g \in G$, the fixedpoint set $F = F(g)$ is a subcomplex of M; (2) the natural cell structure of the orbit space $\mathfrak{M} = M/G$ and the projection $\phi : M \rightarrow \mathfrak{M}$ are simplicial and (3) ϕ maps each simplex homeomorphically and (4) if σ, σ' are oriented simplexes of M, then $\phi\sigma = \phi\sigma'$ implies $\sigma' = g\sigma$ for some $g \in G$.

From the piecewise linear point of view, these conditions are not restrictive. In fact if (G, M) is simplicial, there is an induced action (G, M_1), M_1 a simplicial subdivision of M, which satisfies (1). If (G, M) satisfies (1), it is a straightforward exercise to show that the induced action (G, M''), where M'' is the second barycentric subdivision, satisfies (1), (2), (3), (4).

We shall assume from here on that $G = Z_p$, $p \geq 2$ and $F = F(G)$ is a simple closed curve. From condition (1), F is a polygon, subcomplex of M.

Moise [1] proved

THEOREM 1. *If M is homeomorphic to a euclidean 3-sphere there exists a compact orientable polyhedral 2-manifold Y in M (i.e. piecewise linearly imbedded in M) such that[2] $\partial Y = F$ and such that the p images of $Y - F$ are disjoint.*

Moise showed further that if F is unknotted in the 3-sphere M, then (G, M) is equivalent to a rotation. It is sufficient to prove

THEOREM 2. *If M is homeomorphic to a euclidean 3-sphere and F is unknotted, there exists a manifold Y which has the properties stated in Theorem 1 and is a disc.*

The proof of Theorem 1 in [1] employs a number of special technical devices. We give here an alternative proof which seems shorter and more direct. The same proof in conjunction with Dehn's lemma gives Theorem 2. Theorem 1 will be proved essentially by producing a 2-manifold $\mathcal{C}$ in M/G such that $\partial\mathcal{C} = \mathfrak{F}(=\phi F)$. The required 2-manifold in M is the union of F and a component of $\phi^{-1}(\mathcal{C} - \mathcal{C} \cap \mathfrak{F})$.

If M is oriented and without boundary, and if the induced action $(G, M - F)$ is free, then $\mathfrak{M} = M/G$ *is an oriented manifold without boundary.* For let x be a vertex of M, $\kappa = \phi x$, $W_x = \mathrm{St}(x, M)$ ($=$ star of x in M). Since $\phi \mid M - F$ is a local homeomorphism, one sees that if $x \in M - F$, ϕ maps W_x

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[2] ∂X denotes the boundary of X in the sense of manifold theory. If X is oriented, so is ∂X.