ILLINOIS JOURNAL OF MATHEMATICS
Volume 28, Number 2, Summer 1984

# SOME REMARKS ON A PAPER OF C. DOYLE AND D. JAMES ON SUBGROUPS OF $SL(2, \mathbf{R})$

BY

GERHARD ROSENBERGER

## 1. Introduction

Let $G$ be a subgroup of $SL(2, \mathbf{R})$. We say $G$ is elementary if the commutator of any two elements of infinite order has trace 2; equivalently, $G$ is elementary if any two elements of infinite order (regarded as linear fractional transformations) have at least one common fixed point.

The elementary subgroups of $SL(2, \mathbf{R})$ are well known and easily dealt with (cf. [2; pp. 117–147]).

C. Doyle and D. James [1] proved (in a slightly modified formulation) that a non-elementary subgroup of $SL(2, \mathbf{R})$ can be generated by hyperbolic matrices. In [1] they construct a generating system for a non-elementary group $G$ which contains only hyperbolic matrices. But in general this generating system of $G$ is not minimal. Here we generalize the above result of C. Doyle and D. James and prove that a non-elementary subgroup of $SL(2, \mathbf{R})$ can be generated by a minimal generating system which contains only hyperbolic matrices.

At the end of this note we give some remarks on the other results in the paper of C. Doyle and D. James [1].

## 2. Preliminary Remarks

Let $H$ be any group. We call a cardinal number $r$ the rank $r(H)$ of $H$ if $H$ can be generated by a generating system $X$ with cardinal number $r$ but not by a generating system $Y$ with cardinal number $s$ less than $r$. Let $r(H)$ be the rank of $H$. We call a generating system $X$ of $H$ minimal if $X$ has the cardinal number $R(H)$.

Now let $G$ be a subgroup of $SL(2, \mathbf{R})$. We use the notation $[A, B]$ for $ABA^{-1}B^{-1}$, the commutator of $A$, $B \in G$, and let tr $A$ be the trace of $A \in G$. An element $A \in G$ is called hyperbolic if $|\mathrm{tr}\, A| > 2$; it is called elliptic if $|\mathrm{tr}\, A| < 2$.

Received December 14, 1981.