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FOURIER ANALYSIS AND DETERMINING SETS FOR RADON MEASURES ON $\mathbf{R}^n$

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1. Introduction

Let C be a linear space of Radon measures on $\mathbf{R}^n$. Then we say a bounded Borel set $E \subseteq \mathbf{R}^n$ is a determining set for C if and only if $\mu \in C$, $\mu(x + E) = 0$ for all $x \in \mathbf{R}^n$ implies $\mu \equiv 0$ (equivalently, $\mu, \gamma \in C$ and $\mu(x + E) = \gamma(x + E)$ for all $x \in \mathbf{R}^n$ implies $\mu \equiv \gamma$). The general problem is: given a class of measures C, find conditions under which a given set E is a determining set. In this paper we study this problem for various classes of measures under different growth/decay conditions of the measures at ∞. Let M be the class of all Radon measures, M_T the class of "tempered" Radon measures, M_0 the class of measures "vanishing at ∞" and M_F the class of finite complex measures. Then we have $M_F \subseteq M_0$, $M_T \subseteq M$.

We describe the following interesting features:

(i) No bounded Borel set E is a determining set for M [3].

(ii) No "symmetric" bounded Borel set is a determining set for M_T (Corollary 3.4).

(iii) No "spherically symmetric" bounded Borel set is a determining set for M_0 (Theorem 4.3)

(iv) Every bounded Borel set of positive Lebesgue measure is a determining set for M_F (see [10]).

The problem of finding determining sets when one allows rotations as well as translations is an old one and is known in the literature as Pompeiu's problem [3], [13]. However, in this paper we restrict our attention, for the most part, to determining sets under translations. Some of the results presented here are our own while others rephrase old results in the language of determining sets. The main tool used here is the Fourier transform on $\mathbf{R}^n$. Since the basic question is measure theoretic it would be interesting if we could find geometric proofs of the results obtained without appealing to Fourier analysis (as for example in the proof of Helgason's support theorem for the Radon transform [5]).

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