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ON THE NUMBER OF CHARACTERS IN A p-BLOCK OF A p-SOLVABLE GROUP

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In [3], R. Brauer conjectured that the number $k(B)$ of ordinary irreducible characters of a finite group G in a p-block B is bounded by the order $|D|$ of a defect group D of B. It is fairly easy to show that $k(B) \leqslant |D|^2$, even better, Brauer and Feit [4] showed $k(B) \leqslant \frac{1}{4}|D|^2 + 1$.

The conjecture has been proved under some very specific assumptions on D. If D is cyclic, it follows from Brauer-Dade theory [2], [6].

If D is elementary abelian of order 8, it is true as shown by Landrock in [12], which also contains a review of what little is known in general.

In case of p-solvable groups, Nagao [14] used the method of Fong [8] to reduce the problem to the following question:

Let V be an elementary abelian p-group on which a p'-group G acts faithfully and irreducibly. Is it then true that the number of conjugacy classes of the semidirect product GV is bounded by the order of V?

This sounds very innocent; however, an affirmative answer would give information on all faithful (and irreducible) representations of all finite groups over nearly all finite fields—excluding only those with a characteristic dividing the group order.

The aim of this paper is to develop some ideas how to tackle the problem (§§ 1–4). A key role is played by a generalized character δ of G which measures how far an element $g \in G$ is from acting trivially on V. It turns out that we need information on δ only for a—possibly very small—subgroup of G, namely the centraliser of an arbitrary element $v \in V$.

As an application, it is shown that the answer to the above question is yes, if G is a supersolvable group (§§ 6–7). Also, some consequences for more general classes of finite groups (not just semidirect products) are given. More specifically, Brauer's conjecture holds for p-blocks of solvable groups with a supersolvable p'-Hall group (Theorem 7.4).

The result of § 5 gives a fairly general criterion for an irreducible module to stay irreducible when restricted to the centraliser of an abelian normal subgroup.

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