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NOETHERIAN $\mathbf{Z}_p$ $[[T]]$-MODULES, ADJOINTS, AND IWASAWA THEORY

BY

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Introduction

Let p be a rational prime and set $\Lambda = \mathbf{Z}_p$ $[[T]]$. We define a functor α on the category of noetherian torsion Λ-modules. This map associates to each module X its adjoint $\alpha(X)$ which has no non-trivial finite Λ-submodules. There exist Λ-module homomorphisms $x \to \alpha(x)$ and $\alpha(x) \to x$, each having finite kernel and cokernel. These maps are *not* canonical but there is a canonical homomorphism from x to $\alpha(\alpha(x))$, again having finite kernel and cokernel.

Let k_n be the nth layer of the basic $\mathbf{Z}_p$-extension k_∞ of a number field k. Fix disjoint finite sets S and R of places of k with S containing all the nonarchimedian places and R containing no primes above p. By A_n we signify the p-part of the R-generalized S-class group of k_n. Set

$$H = \operatorname{Hom}_{\mathbf{Z}_p}\left(\varinjlim A_n, \mathbf{Q}_p/\mathbf{Z}_p\right)$$

where the limit is with respect to the natural maps induced by extension of S-ideals. H has a natural structure as $\mathbf{Z}_p[G(k_\infty/k)]$-module and we identify $\mathbf{Z}_p[G(k_\infty/k)]$-modules and Λ-modules by requiring that $(T+1)$ act as a topological generator γ of $G(k_\infty/k)$. Thus H is a Λ-module. We show that H may be interpreted as an adjoint. In particular, it is noetherian.

Let $A = \varinjlim A_n$ where the limit is with respect to the norm maps. We show that A is pseudo-isomorphic to H and interpret A as a Galois group. Our study of H and A depends on class field theory and the results may be thought of as analogs of the Artin isomorphism for A_n.

The exposition of this paper is based on Iwasawa's presentation in a course at Princeton during Spring, 1980. It seemed useful for us to present it here since the results are discussed only briefly in the extant literature. Our new contribution is to work with R-generalized S-class groups. Iwasawa only considered the usual class groups.

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