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ERGODIC THEOREMS FOR CONVOLUTIONS OF A MEASURE ON A GROUP

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Introduction

Let G be a Hausdorff locally compact group (called a group here) and let μ be a probability measure in $M(G)$, the finite regular Borel measures on G. By $\|\mu\|_1$, we will denote the total variation norm of $\mu \in M(G)$. Suppose that (X, β, m) is a measure space with m being a σ-finite positive measure. Let T be a representation of G as invertible measure-preserving transformations of (X, β, m). Then there is an operator on $L_2(X, \beta, m)$ associated with μ, denoted by T_μ, which integrates T_g, $g \in G$, with respect to μ. This operator can be defined weakly by

$$\langle T_\mu f_1, f_2 \rangle = \iint f_1(T_{g^{-1}}x)\overline{f_2(x)}\, dm(x)\, d\mu(g)$$

for all $f_1, f_2 \in L_2(X, \beta, m)$. In the books by Tempelman [34], [35] and in several recent articles (see Bellow, Jones, and Rosenblatt [3], [4], [5], Derriennic and Lin [9], and Rosenblatt [28, 29]) in the case of probability measures μ, the norm and almost everywhere behavior of the iterates of T_μ on $L_p(X, \beta, m)$ have been studied with some success. In this article, these various results are extended to general locally compact groups, including a specific discussion of the influence that the spectral behavior of μ and T_μ have on the conclusions. Various positive results about norm and a.e. convergence of the iterates of T_μ will be obtained, and counterexamples will be discussed which illustrate the limitations on the theorems and the techniques that are used.

1. Direct integral formulas

The first issue is to clarify the definition of $T_\mu = \int T_g\, d\mu(g)$. If $d\mu = \phi\, d\lambda_G$, where λ_G is a right-invariant Haar measure on G, and $\phi \in L_1(G, \lambda_G)$, then this operator is the standard integration of the representation T as unitary

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