TWIST-SPINNING SPHERES IN SPHERES

BY

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I. Introduction

Let $\theta^{m,n}$, $n > 4$, denote the group of h-cobordism classes of pairs of spheres (S^m, Σ^n), where S^m denotes an m-sphere with its usual structure and Σ^n denotes an embedded n-sphere which may have an exotic structure, [2], [9].

Our aim is to introduce an operation, which will be called *twist-spinning*;

$$\phi : \theta^{m,n} \times \pi_l(SO(n) \times SO(m - n)) \to \theta^{m+l,n+l}.$$

When $m = n + 2$, the operation is twist-spinning as defined by Artin–Zeeman [1], [12], except that we have introduced tangential twisting by elements of $\pi_l(SO(n))$. The operation restricted to the embedded sphere Σ^n of the pair (S^m, Σ^n) is equivalent to a pairing of Milnor–Munkres [5], [6] (also Novikov [7]), except that the group $\pi_l(SO(n - 1))$ has been replaced here by $\pi_l(SO(n))$. Another operation may be defined by replacing $\theta^{m,n}$ by $I^{m,n}$ the group of regular homotopy classes of immersions of S^n in S^m.

In §2, the operation is described and defined. In §3 it is related to a relative version of the Milnor–Munkres–Novikov pairing;

$$\pi_0(\mathrm{Diff}_C\,(R^{m-1}, R^{n-1})) \otimes \pi_l(SO(n - 1) \times SO(m - n)) \to \pi_0(\mathrm{Diff}_C\,(R^{m+l-1}, R^{n+l-1})).$$

The resulting operation on normal bundles is investigated in §4 and found to be related to the Whitehead product pairing,

$$\pi_n\, BSO(m - n) \otimes \pi_{l+1}\, BSO(m - n) \to \pi_{n+l}\, BSO(m - n).$$

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II. Twist-spinning

Let (S^m, Σ^n) we a pair of spheres representing an element of $\theta^{m,n}$. Let D^m_+, D^m_- denote the upper and lower hemispheres of S^m respectively. Then $S^m = D^m_+ \cup D^m_-$. Now the pair (S^m, Σ^n) is diffeomorphic to

$$(D^m_+ \cup D^m_-,\, D^n_+ \cup \Sigma^n - \mathrm{Int}\, D^n_+)$$

where D^n_+ is a disc embedded in Σ^n and the inclusion $D^n_+ \subset D^m_+$ is assumed to be standard; further we may suppose the inclusion $\Sigma^n - \mathrm{Int}\, D^n_+ \subset D^m_-$ coincides with the standard inclusion $D^n_- \subset D^m_-$ near the boundary of $\Sigma^n - \mathrm{Int}\, D^n_+$.

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