ABSOLUTE EQUIVALENCE OF EXTERIOR DIFFERENTIAL SYSTEMS

BY

H. H. JOHNSON[1]

This paper concerns an equivalence relation first defined by E. Cartan for certain systems of ordinary differential equations [2]. He called two systems absolutely equivalent if they had isomorphic prolongations. A similar concept was used in Cartan's theory of infinite groups [1]. We extend Cartan's definition to general exterior differential systems. For ordinary equations one has only normal prolongations, but in general it is necessary to define and study partial prolongations. This is done and absolute equivalence is defined in §1. In §2 an integer is found which is an absolute invariant and which may be calculated from any given involutive system. In §3 other invariants are found which are calculated from the sequence of normal prolongations of a system. Examples are given. All manifolds, functions and forms are complex analytic. We use the notations and definitions of [4]. We also deal only with systems which are *involutive* at each of their points.

1. Absolute equivalence

Let D_p and M be neighborhoods of 0 in $R^p = \{(x_1, \cdots, x_p)\}$ and $R^m = \{(y_1, \cdots, y_m)\}$, respectively. Let $D = D_p \times M$ Then $J^k(D_p, M)$ denotes the manifold of k-jets of maps on D_p into M. The usual source and target projections are α and β, and

$$\rho = \alpha \times \beta : J^k(D_p, M) \to D.$$

If $f : D_p \to M$, let $\tilde{f} : D_p \to D$ be defined by $\tilde{f}(x) = (x, f(x))$. Let Ω be the module of 1-forms generated by $dx_1, \cdots, dx_p$ on D_p. We shall consistently use the same notation Ω for $\alpha^*\Omega$ on D or $J^k(D_p, M)$. Let (Σ, Ω) be an exterior differential system on D having independent variables Ω. Denote by $(P^k\Sigma, \Omega)$ the k^{th} prolongation of (Σ, Ω) on $J^k(D_p, M)$.

If $D' = D_p \times M'$ is a submanifold of D by an imbedding F such that $\alpha F = \alpha$, and if $F(D')$ contains the manifold of integral points of (Σ, Ω), the restriction of (Σ, Ω) to D' is called an *admissible restriction* of (Σ, Ω).

If $F : D \to D$ and $f : D_p \to D_p$ are bi-analytic functions such that $\alpha F = f\alpha$, then the transformed system $(F^*\Sigma, f^*\Omega)$ is said to be a *transform* of (Σ, Ω).

DEFINITION 1. A system (Σ_1, Ω) on $D_1 = D_p \times M_1$ is a *partial prolongation* of (Σ, Ω) on D if there exist maps

$$a : D_1 \to D \quad \text{and} \quad b : J^1(D_p, M) \to D_1$$

which satisfy:

Received March 4, 1965.

[1] Supported by an Office of Naval Research Contract.