A KNOTTED CELL PAIR WITH KNOT GROUP Z

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In [2], L. C. Glaser and I proved that a locally flat cell pair (C, C') of type (n, k) was unknotted if $n - k$ was either 1 or was greater than 2, $n \geq 4$. We also proved that if $n - k = 2$ and both $C - C'$ and $\text{Bd}\, C - \text{Bd}\, C'$ have the homotopy type of the 1-sphere, then (C, C') is unknotted, $n \geq 6$. Theorem 3 of this paper gives examples of cell pairs (C, C') of type $(n, n - 2)$, $n \geq 6$, such that $\pi_1(C - C') = Z$, but $\pi_1(\text{Bd}\, C - \text{Bd}\, C') \neq Z$.

It should be noted that the results obtained here can also be obtained in a piecewise linear or differentiable setting rather than the locally flat setting. In proving Theorem 4 in the differentiable case, one must apply the so-called "smoothing the corners" process; otherwise the proofs are not significantly different from what is done here.

Let C be an n-cell and C' be a k-cell, then (C, C') is called a cell pair of type (n, k) if C' is a spanning cell of C; that is, the boundary of C' is contained in the boundary of C and the interior of C' is contained in the interior of C. The boundary of a cell D is denoted by $\text{Bd}\, D$. A cell pair, or a sphere pair, is called unknotted if it is homeomorphic to the appropriate standard cell pair, or standard sphere pair. Finally, let (E^n, E^k) and (E^n_+, E^k_+) denote the standard Euclidean space pair of type (n, k) and the standard closed Euclidean half-space pair of type (n, k) respectively. A manifold pair (W, W') is called locally flat if each point w of W' has a neighborhood homeomorphic to (E^n, E^k) or (E^n_+, E^k_+) according to whether w is in the interior of W' or on the boundary of W'.

THEOREM 1 (Hudson and Sumners, see Cor. 2 of [3]). *For $n \geq 4$ there exists a locally flat sphere pair (S, S') of type $(n, n - 2)$ such that*

(1) *(S, S') is unknotted,*

(2) *$S = E \cup F$ n-cells with $\text{Bd}\, E = \text{Bd}\, F = E \cap F$,*

(3) *$S' = E' \cup F'$ where E' and F' are locally flat spanning $(n - 2)$-cells of E and F respectively, and*

(4) *$\pi_1(E - E') \neq Z$ and $\pi_1(F - F') \neq Z$.*

As stated above, this theorem was proved by J. F. P. Hudson and D. W. L. Sumners in [3]. They gave a method of constructing such cell pairs. We suggest here an alternative construction. This method, using twist spinning,

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