WALSH SERIES AND ADJUSTMENT OF FUNCTIONS ON SMALL SETS

BY

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1. Introduction

D. E. Menshov proved that a measurable function finite almost everywhere on $[0, 2\pi]$ can be changed on a set of measure less than ε to a function whose Fourier series converges uniformly ([4]; see also [1, Chapter VI]). Recently, B. D. Kotlyar [3] proved an analogous theorem for Walsh series. Menshov proved also that for continuous functions the set where the adjustment is made can be chosen to depend only on ε and the modulus of continuity.

In this paper, we present a different proof of Kotlyar's theorem that contains also an analogue of Menshov's theorem on continuous functions. Actually, our result contains somewhat more. Let $\{p_\nu\}$ and $\{q_\nu\}$ be increasing sequences of positive integers such that

$$p_1 < q_1 < p_2 < q_2 < \cdots, \qquad \{q_\nu/p_\nu\} \quad \text{is unbounded.} \tag{1}$$

Define

$$W = \bigcup_{\nu=1}^{\infty} [\psi_k : p_\nu \leqq k < q_\nu] \tag{2}$$

where ψ_k is the k-th Walsh function. We shall prove that a measurable function can be changed on a small set to a function whose Walsh-Fourier series converges uniformly and contains only Walsh functions in W.

THEOREM. *Let f be measurable and finite almost everywhere on* $[0, 1]$ *and let a positive ε be given. Then there exists a function g such that*

(a) *$g(x) = f(x)$ except on a set E of measure less than ε,*

(b) *the Walsh-Fourier series of g contains only Walsh functions in the set W defined by* (1) *and* (2) *and converges uniformly.*

Furthermore, suppose $\rho(\delta)$ is a nondecreasing function defined for $\delta > 0$ with

$$\lim_{\delta\to 0} \rho(\delta) = 0.$$

Then there is a set E depending only on ε and ρ such that (a) *and* (b) *hold for every continuous function f whose modulus of continuity $\omega(\delta)$ satisfies*

$$\omega(\delta) \leqq \rho(\delta).$$

This theorem contains a previous result of the author [5].

COROLLARY. *The system of Walsh functions W defined by* (1) *and* (2) *is total in measure on* [0, 1].

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