# MODULAR SUBGROUPS OF FINITE GROUPS II

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The subgroup $M$ of the group $G$ is said to be *modular in* $G$ ($M \mathfrak{m} G$) if

$$(U \cup M) \cap V = U \cup (M \cap V) \quad \text{for all } U, V \subseteq G \text{ such that } U \subseteq V,$$

and

$$(U \cup M) \cap V = (U \cap V) \cup M \quad \text{for all } U, V \subseteq G \text{ such that } M \subseteq V.$$

In [5] we proved among other results that $M/M_G$ is nilpotent and $M^G/M_G$ is supersolvable for a modular subgroup $M$ of a finite group $G$ ($M_G$ being the core, $M^G$ the normal closure of $M$ in $G$). One of the problems that remained open in [5] was to discover the exact structure of $G/M_G$, $M^G/M_G$, and $M/M_G$. In the present paper we solve this problem modulo the quasinormal Sylow subgroups of $M/M_G$. We prove the following

THEOREM. *Let $M$ be modular in the (finite) group $G$, and let $Q/M_G$ be a $q$-Sylow subgroup of $M/M_G$ which is not quasinormal in $G/M_G$, $q$ a prime.*

*Then $G/M_G = Q^G/M_G \times K$, where $Q^G/M_G$ is a $P$-group of order $p^n \cdot q$, $p$ a prime, $p > q$, and $(|Q^G/M_G|, |K|) = 1$.*

(For the definition of a $P$-group see [6, p. 12] or [5].)

An immediate consequence of this theorem is the following

COROLLARY. *Let $M$ be modular in $G$, and let $M_G = 1$* (to make notation simpler).

*Then $G = P_1 \times \cdots \times P_r \times K$, where $P_i$ is a $P$-group of order $p_i^{n_i} \cdot q_i$, $p_i$, $q_i$ primes, $p_i > q_i$, $(|P_i|, |P_j|) = (|P_i|, |K|) = 1$ $(i, j = 1, \cdots, r; i \neq j)$, and where $M = Q_1 \times \cdots \times Q_r \times (M \cap K)$, with $Q_i$ being a $q_i$-Sylow subgroup of $P_i$, and $M \cap K$ being quasinormal in $G$.*

This corollary gives the solution of the problem mentioned above modulo the quasinormal part $M \cap K$ of $M$, about which we cannot say very much (except, of course, that it is quasinormal in $G$). Since it is obvious that a subgroup $Q$ is modular in a group $G$ whenever $G/Q_G$ has the structure given in the Theorem, we also cannot say anything about the structure of the complement $K$ in $G/M_G$.

Some other consequences of the theorem are perhaps worth mentioning.

(1) *Let $M$ be modular in $G$, and let $Q/M_G$ be a Sylow subgroup of $M/M_G$. Then $Q$ is modular in $G$.*

(2) *A minimal modular (but not normal) subgroup of a group is of prime power order.*

(3) *Let $M$ be modular in $G$, and let $q$ be a prime dividing $|M : M_G|$. Then there is a normal subgroup $N$ of index $q$ in $G$.*

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