APPLICATION OF DE BRANGES SPACES OF INTEGRAL FUNCTIONS TO THE PREDICTION OF STATIONARY GAUSSIAN PROCESSES

BY

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Usage

Greek letters α, β, γ, etc. stand for complex numbers. * indicates conjugation. $f^\times$ means $[f(\gamma^*)]^*$. $\lg^+ x$ is the logarithm of the bigger of 1 and x, and $\lg^- x = \lg^+ (1/x)$ so that $\lg x = \lg^+ x - \lg^- x$. $\int$ indicates integration over the line R^1, as in

$$\int (1 + \gamma^2)^{-1} \lg^+ |f| = \int_{-\infty}^{+\infty} (1 + \gamma^2)^{-1} \lg^+ |f(\gamma)| \, d\gamma.$$

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