NORMAL FIBRATIONS FOR COMPLEXES[1]

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0. Introduction

Suppose a smooth manifold N is embedded in another M. Then there is the familiar notion of the normal bundle ξ of the embedding. However, one can regard this bundle as a spherical fibration. And it is easily seen that if T is a tubular neighborhood of N, then if we replace the map $\partial T \subseteq T$ by a fibration ξ', (which we view as a fibration over N), ξ' is fiber-homotopically equivalent to ξ. If one forgets about the smoothness (or PL) structure of N, and demands that it merely be an embedded Poincaré duality complex, then the same construction, according to Spivak, still yields a spherical fibration of the appropriate dimension, although it may no longer be fiber-homotopically equivalent to a bundle. (Here, we replace "tubular neighborhood" by "regular neighborhood"). In fact, we do not need to have an actual geometric embedding of N in M; it will suffice that a "thickening" $\bar{N}$ of N is a codimension 0 submanifold of M.

In this paper, we altogether abandon any conditions on N other than that it be of the homotopy type of a finite complex. One can then perform the same sort of construction, i.e. take a thickening $\bar{N}$ and look at the result of replacing $\partial\bar{N} \subseteq \bar{N}$ by a fibration ν.

What makes this interesting is that the stable homotopy type of the fiber of this fibration depends only on N and not at all on the thickening, thus generalizing the situation for Poincaré complexes. Moreover, if one suspends a thickening, then the fibration associated with the suspension is the suspension of the fibration associated with the original thickening. One can then ask questions about the fibrations to derive information about the thickening.

In particular, one shows that desuspending the fibration is roughly equivalent to desuspending the thickening. In the metastable range, in fact, one can restrict one's attention to the question of whether sections exist. This leads, among other things, to a generalization of Hirsch's compression theorem [l] in the metastable range. These results are exposited in the first three sections, along with some preliminary remarks on fibrations.

Sections 4, 5, and 6 utilize the notion of normal fibration of a thickening to get various results about Poincaré-duality spaces, among other things. It is shown, for instance, that one can talk about thickenings of a complex into Poincaré-duality pairs, and that the resulting theory greatly resembles that exposited by Wall [e] in the smooth and PL case. We also give a new proof

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