SERIES EXPANSIONS AND INTEGRAL REPRESENTATIONS OF GENERALIZED TEMPERATURES

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1. Introduction

The generalized heat equation is given by

$$\frac{\partial^2 u}{\partial x^2} + \frac{2\nu}{x}\frac{\partial u}{\partial x} = \frac{\partial u}{\partial t}, \tag{1.1}$$

ν a fixed positive number. The fundamental solution of (1.1) is the function $G(x; t) = G(x, 0; t)$, where

$$\begin{aligned} G(x, y; t) &= \int_0^\infty e^{-u^2 t}\mathcal{J}(xu)\mathcal{J}(yu)\, d\mu(u), \quad t > 0, \\ &= \left(\frac{1}{2t}\right)^{\nu+1/2} \exp\left(-\frac{x^2+y^2}{4t}\right)\mathcal{I}\left(\frac{xy}{2t}\right), \end{aligned} \tag{1.2}$$

with

$$d\mu(u) = \frac{2^{1/2-\nu}}{\Gamma(\nu+\frac{1}{2})} u^{2\nu}\, du,$$

$$\mathcal{J}(z) = 2^{\nu-1/2}\Gamma(\nu+\tfrac{1}{2})z^{1/2-\nu}J_{\nu-1/2}(z), \quad \mathcal{I}(z) = 2^{\nu-1/2}\Gamma(\nu+\tfrac{1}{2})z^{1/2-\nu}I_{\nu-1/2}(z),$$

$J_\alpha(z)$ being the ordinary Bessel function of order α and $I_\alpha(z)$ the Bessel function of imaginary argument. It is well known (see [5]) that if $u(x, t)$ is a solution of (1.1), so is its Appell transform $u^A(x, t)$ defined by

$$u^A(x, t) = G(x; t)u(x/t, -1/t). \tag{1.3}$$

The Poisson-Hankel transform of a function φ is given by

$$\int_0^\infty G(x, y; t)\varphi(y)\, d\mu(y), \qquad t > 0, \tag{1.4}$$

whenever the integral exists. Taking $\varphi(x) = x^\gamma$, we set

$$S_{\gamma,\nu}(x, t) = \int_0^\infty y^\gamma G(x, y; t)\, d\mu(y), \qquad \gamma > -2\nu. \tag{1.5}$$

$S_{\gamma,\nu}(x, t)$ satisfies equation (1.1), and in particular, if $\gamma = 2n$, $S_{\gamma,\nu}(x, t)$ is the generalized heat polynomial $P_{n,\nu}(x, t)$ studied in [5]. In that paper, those solutions of (1.1) were characterized which have representations in series of $P_{n,\nu}(x, t)$ and of their Appell transforms $W_{n,\nu}(x, t)$. It is the present

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