LEBESGUE SPACES OF PARABOLIC POTENTIALS

BY

RICHARD J. BAGBY

Introduction

We define a class of spaces $\mathcal{L}_\alpha^p$ via Fourier transform techniques. These spaces have been studied previously by Sampson [11]. They arise in the study of the heat equation; they are the parabolic analogue of the spaces of Bessel potentials introduced by Aronszajn and Smith [1] and by Calderón [4]. The results obtained in this paper are analogous to results obtained by Strichartz [13] for Bessel potentials.

The first chapter contains the basic facts about $\mathcal{L}_\alpha^p$ spaces. In the second chapter we characterize some of these spaces in terms of an integral norm of a difference quotient. We develop an interpolation theory for these spaces in the third chapter. These results are of some interest in themselves; they are used in the fourth chapter to find sufficient conditions for the product of two functions to be in one of the spaces $\mathcal{L}_\alpha^p$.

Establishing the characterization of Chapter 2 requires a number of calculations. The appendix contains the worst of these.

This paper consists essentially of the author's doctoral dissertation at Rice University. I wish to thank my advisor Dr. B. Frank Jones for his help. Financial support was provided by the United States Air Force, N.A.S.A., and the Schlumberger Foundation.

1. Preliminaries

1.1 *Notation.* Let E^{n+1} denote Euclidean $(n+1)$-space. Points in E^{n+1} will be denoted in the form (x, t), where $x \in E^n$. Unless explicitly stated otherwise, all function spaces are assumed to be spaces of functions defined on E^{n+1}.

The usual inner product in E^n will be denoted by $x \cdot y$. For $x \in E^n$, $|x| = (x \cdot x)^{1/2}$. Differential operators are expressed in the form

$$D_x^\alpha D_t^j = (\partial/\partial x_1)^{\alpha_1} \cdots (\partial/\partial x_n)^{\alpha_n} (\partial/\partial t)^j;$$

the order of the multi-index α is denoted by $|\alpha| = \alpha_1 + \alpha_2 + \cdots + \alpha_n$. The Laplace operator in E^n is denoted by Δ_x.

Let $\mathcal{S}$ denote the space of C^∞ functions ϕ satisfying

$$\sup_{(x,t)} |P(x, t) D_x^\alpha D_t^j \phi(x, t)| < \infty$$

for any polynomial P and any α, j. $\mathcal{S}$ is given the usual topology; see Schwartz [12]. The dual of $\mathcal{S}$ is denoted by $\mathcal{S}'$; its elements are called tempered distributions.

Received April 15, 1969.