MOORE-POSTNIKOV SYSTEMS FOR NON-SIMPLE FIBRATIONS

BY

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In [9], Moore outlined a method of factorizing a fibration into a sequence of fibrations whose fibres are Eilenberg-MacLane spaces. Several accounts of this theory have since appeared [e.g. 4, 6], but these treat only the case when the fibration is simple in that the fundamental group of the base acts trivially on the homotopy groups of the fibre. This is untrue for certain interesting fibrations, such as non-orientable sphere-bundles. This paper develops the theory without the hypothesis of simplicity.

Our main theorem (3.4) states that a fibration with fibre $K(\pi, n-1)$ is completely determined (up to fibre homotopy equivalence) by a 'characteristic class' in the n^{th} cohomology group of the base with suitably twisted coefficients π. We prove this by constructing a classifying space $\hat{K}(\pi, n)$ for fibrations whose fibres have the homotopy type of $K(\pi, n-1)$ [cf. 10]. We define "representable" cohomology with twisted coefficients in terms of the space $\hat{K}(\pi, n)$, and reconcile our definition with a classical one. The spaces $\hat{K}(\pi, n)$ seem to be useful objects in non-simple obstruction theory. The corresponding semi-simplicial complexes have been considered by Gitler [3].

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1. The complex $\hat{K}(\pi, n)$

All our spaces will have compactly-generated Hausdorff topologies, in accordance with the system described by Steenrod [11]. In a topological group G, the multiplication map $G \times G \to G$ is required to be continuous only when $G \times G$ has the compactly-generated topology. Except where the reverse is stated, all spaces have basepoints (denoted by o) and all maps and homotopies are based. We abbreviate "(X, A) has the homotopy type of a CW pair" to "(X, A) is of CW type", etc.

1.1. Let π be an abelian group, and aut π its group of (left) automorphisms. Take an Eilenberg-MacLane CW-complex $K(\pi, n)$ which is a topological abelian group on which aut π acts by cellular automorphisms. For instance, the Milnor realization of the standard semi-simplicial complex $K(\pi, n)$ [2, 7] will do. Let Q be a CW-complex of type $K(\text{aut } \pi, 1)$. Then the universal cover $\tilde{Q}$ is a contractible complex on which aut π acts freely and cellularly on the left. The diagonal action of aut π on $K(\pi, n) \times \tilde{Q}$ (given by

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