TOPOLOGIES IN LOCALLY COMPACT GROUPS II

BY

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Introduction

In this paper we study the partially ordered set of locally compact group topologies on a given abelian group. Our main interest is the cardinality of a given interval $[a, b]$ in this set. We prove that $| [a, b] | \geq c$ or is finite. This generalises the results obtained in [4] and [5] and also answers a question raised in [5]. Our methods involve delicate ways of embedding R^n in a compact group. These embedding theorems are given in Section 1. We have to study a relation $\sim$ in the set of subgroups of a given torsion free abelian group. This notion resembles that of quasi-isomorphism used by Beaumont and Pierce [1]. This is done in Section 2. We make heavy use of the results proved in [4] and [5].

Notation. All groups considered in this paper are abelian. All topological spaces considered are Hausdorff. The notions and terminologies on topological groups are as in [3] in general. T denotes the circle group with usual topology and multiplication. If G is a topological group, we say G is T-free if T is not a topological summand of G. Similarly G is said to be Z-free if G does not have Z as an algebraic summand. If G is a group and $a_1, a_2, \cdots\cdots, a_n$ are elements of G then $[a_1, \cdots, a_n]$ denotes the sub-group generated by $a_1, a_2, \cdots, a_n$ in G. Isomorphism (topological) of two groups (topological) G_1, G_2 is denoted by $\approx$.

Isomorphic embeddings of R^n into a compact abelian group G

LEMMA 1.1. *Let H be a subgroup of $R^n (n \geq 1)$. Let $\bar{H} = F \oplus V$ where V is a subspace of R^n and F is a free group different from $\{0\}$. Then Z is a summand of H.*

Proof. Follows from standard arguments and the structure of closed subgroups of R^n.

LEMMA 1.2. *Let $\hat{G}$ be a torsion free group of rank n $(n \geq 1)$. Then $\hat{G}$ can be embedded as a dense subgroup of R^n by a group isomorphism if and only if $\hat{G}$ is Z-free. When $\hat{G}$ is Z-free, we can obtain such an embedding as follows: Choose a maximal independent set $(a_1, \cdots, a_n)$ in $\hat{G}$ over the integers. Define a map*

$$\phi_0 : \{a_1, \cdots, a_n\} \rightarrow R^n$$

arbitrarily except that the set $\{\phi_0(a_1), \cdots, \phi_0(a_n)\}$ generates R^n over R. Using

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