ON ABSOLUTELY CONTINUOUS TRANSFORMATIONS FOR MEASURE SPACES[1]

BY

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1. Introduction

If T is a transformation from a first measure space (S, Σ, μ) onto a second measure space (S', Σ', μ'), and one wishes to transform a problem involving integration in the second space to an equivalent problem in the first space, he usually introduces a weight function W' which assigns to each point s' in S' a value reflecting its importance relative to the image under T of a subset D of S which belongs to a class $\mathfrak{D}$ having suitable properties. For the case when W' is non-negative, quite general solutions to this question are known [12], [14], [1], [3]. Brooks [2] introduced Banach-valued weight functions; as a special case, signed weight functions may now be used when the spaces are oriented. In this case it is natural to try to express such functions as the difference of two non-negative weight functions [12], [11]. One obvious approach is to consider the total variations V', together with the positive and negative variations V'_+ and V'_-, of W' relative to $\mathfrak{D}$ and to seek conditions which insure that these are non-negative weight functions for which the relations $V' = V'_+ + V'_-$, $W' = V'_+ - V'_-$ hold almost everywhere. This approach is explored in [5], where it is shown that if W' satisfies suitable relations uniformly with respect to $\mathfrak{D}$, then such Jordan-type decompositions do exist for W'. Each weight function W' determines a weight W by the relation

$$W(D) = \int_{S'} W'(\cdot, D)\, d\mu'.$$

Chaney [9] observed that when T is absolutely continuous with respect to these weights, it is possible to determine the weight function W' from the weights W. This suggests that in the absolutely continuous case, for a signed weight function W' one might consider the variations of the weights W, use the result of Chaney to associate weight functions with these and thus obtain a Jordan-type decomposition of the weight function W'. This approach is studied below. Because of the differences in the hypotheses and in the approach, the results obtained here are neither included in nor include the results obtained earlier by the authors in [5]. Our results lead to a transformation formula for signed weight functions under more general conditions than any heretofore known.

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