MORE NEW INTEGER PAIRS FOR FINITE HJELMSLEV PLANES

BY

DAVID A. DRAKE[1]

Introduction

Every finite projective Hjelmslev plane H possesses two integer invariants denoted by t and r. (See, e.g., [4].) Every point of H possesses precisely t^2 neighbor points; r denotes the order of the projective plane canonically paired to H. Such an H will be called a (t, r) *PH*-plane. In [3], Drake and Lenz constructed the first examples of *PH*-planes with invariants (t, r), t not a power of r. These *PH*-planes were so constructed as to possess given *PH*-planes as epimorphic images. The construction methods devised were most successful when the given epimorphic images were taken to be 2-uniform *PH*-planes; i.e., *PH*-planes with $t = r$. In this note, we refine the methods of [3] to obtain *PH*-planes as preimages of strongly n-uniform *PH*-planes for arbitrary n. (See [1].) *PH*-planes with invariants $(t, 2)$ are now known to exist for 41 values of t less than or equal to 1,000 (16 of the 41 thanks to the results of this note); they are known not to exist for three values of t but remain in doubt for the other 956 possible values.

1. Preliminaries

Of great importance to our construction is the following familiar lemma couched in unfamiliar terms. (See [5] or [6].)

THEOREM 1.1 (König's Lemma). *Let T be a tactical configuration with block size and replication number both equal to $r < \infty$. Let $S = \{n_1, \dots, n_r\}$ be a labeling set. Then there is a function f from the flags of T to S such that each point and each line of T occurs in r flags labeled by each of the r elements of S.*

We refer the reader to [1] for the definitions of n-uniformity and strong n-uniformity. If H is a strongly n-uniform *PH*-plane, then the invariants of H are (t, r) where $t = r^{n-1}$. We write $P\ (\sim i)\ Q$ if P and Q are joined by at least r^i lines, $0 \leq i \leq n$; $P\ (\simeq i)\ Q$ if P and Q are joined by exactly r^i lines, $0 \leq i < n$; $P\ (\simeq n)\ Q$ if $P = Q$. By [1, Definition 3.3, Proposition 3.3(2), and Proposition 2.2(2)], we may define $(\sim i)$ and $(\simeq i)$ dually for lines; i.e., $g\ (\sim i)\ h$ if $|g \cap h| \geq r^i$ and $g\ (\simeq i)\ h$ if $|g \cap h| = r^i$. The following result is part of [1, Proposition 2.2] together with [1, Proposition 3.6].

Received March 21, 1975.

[1] The author wishes to acknowledge the hospitality of the Technische Hochschule Darmstadt as well as the financial support of both the Alexander von Humboldt Foundation and the University of Florida (the latter by means of a Faculty Development Grant).