AN HARMONIC ANALYSIS FOR OPERATORS I: FORMAL PROPERTIES

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1. Introduction

Harmonic analysis deals with objects defined on a group or associated with a group and attempts to represent these objects in terms of objects which behave simply with respect to group translation. For example, in the harmonic analysis of functions on compact commutative groups, the functions which behave simplest with respect to translation are the characters, for these, and their scalar multipliers, are precisely the eigenfunctions of the translation operators.

In this paper we initiate a study of a harmonic analysis for operators on homogeneous Banach spaces on the circle group $\mathbf{T}$. In this case the simple operators will be those which commute with translation (we shall call these *invariant operators*) and also those operators which are the composite of an invariant operator and multiplication by a character of $\mathbf{T}$. These simple operators are precisely the operators T which satisfy, for some integer n a functional equation of the form

$$TR_t = e^{int}R_tT, \quad t \in \mathbf{T},$$

where R_t is the translation operator on $\mathbf{T}$ defined by

$$(R_tf)(s) = f(s - t), \quad s \in \mathbf{T}.$$

The operators we call *invariant* are those which are usually called *multipliers* (see [5]) because they are obtained by multiplication on the Fourier transform. In order to avoid confusion, we shall not use this terminology since we shall also be dealing with operations of multiplication by a function on $\mathbf{T}$.

Although we state and prove our results for the circle group $\mathbf{T}$, analogues of all of the results of Sections 2 through 5 are valid for any compact abelian group.

Some of the results we prove here were announced in [2].

2. Homogeneous spaces: invariant, almost invariant, and simple operators

We shall deal with operators acting on a space of functions on the circle group $\mathbf{T}$. The spaces we shall consider will be general enough to include all of the classical Banach function spaces on $\mathbf{T}$.

Received March 14, 1975.

[1] This research was supported in part by a National Science Foundation grant.