THE QUESTION OF EQUIVALENCE OF PRINCIPAL AND COPRINCIPAL SOLUTIONS OF SELF-ADJOINT DIFFERENTIAL SYSTEMS

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1. Introduction

A useful result in the study of differential systems of the form

$$U' = AU + BV, \quad V' = CU - A^*V \tag{1.1}$$

is that, in certain cases, the system (1.1) is disconjugate in some neighborhood of ∞ if and only if the obverse system [2, p. 173]

$$U' = -A^*U + CV, \quad V' = BU + AV \tag{1.2}$$

is disconjugate in some neighborhood of ∞. Given certain assumptions of a variational nature, Reid [4] has established that system (1.1) is disconjugate in a neighborhood of ∞ if and only if there exists a solution which is principal at ∞. Therefore, in those cases where both sets of hypotheses hold, system (1.1) has a principal solution at ∞ if and only if system (1.2) has a principal solution at ∞. The question considered in [2] was that of when a principal solution $(U_\infty; V_\infty)$ gives rise to a principal solution $(U_1; V_1) = (V_\infty; U_\infty)$ of (1.2), i.e., in the terminology used there, the question of when a principal solution is also coprincipal. The present paper gives a number of conditions which are equivalent to the condition of a solution being principal at ∞ if and only if it is coprincipal at ∞. These conditions involve limit type behavior for every conjoined basis. The statements of these conditions which are given in Section 4 are unchanged in the two cases of B and C having either the same or opposite signs for their associated quadratic forms. The coefficients B and C may be singular if certain "normality" assumptions are included. Whereas the previous work primarily concerned the case where B and C were of "opposite" signs, the present study is directed at the case where B and C are of the "same" sign, although, as mentioned, much of the work applies to both cases. As an example of the type of results obtained, limit information is obtained for $U^{-1}V^{*-1}$ in either case for $(U; V)$ an antiprincipal solution. This result is analogous to a result on principal solutions obtained by Reid in the case of B and C nonnegative. Combining these results leads to the conclusion that a large class of such equations has the property that principal solutions and coprincipal solutions coincide.

The primary results of this paper are consequences of combining the following observations. First, the distinguished solution W_∞ of the Riccati equation

$$W' = C - A^*W - WA - WBW, \tag{1.3}$$

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