SPLITTING CERTAIN SUSPENSIONS VIA SELF-MAPS

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The essential content of this paper is a potpourri of examples concerning the splitting of certain suspensions into wedges. Our first example is an improvement of Snaith's [12] stable decomposition of $\Omega^n\Sigma^n X$, $1 < n < \infty$. Here we must work harder than Snaith, but we get a finer splitting (sometimes!). Our second example is provided by the suspension of an H-space. An application is made to the suspension of $SF(2)$ which is in the same spirit as Snaith's decomposition. As a final example we show that a geometric analogue of the classical algebraic transfer for finite covers may, in nice cases, be defined after a single suspension. As an application of these results we use Flynn's calculations [5] to give relatively instant generalizations of Kamata's calculations [8] in [4].

The motivation for describing these three examples in the same paper is that we use the same method throughout. This method, which is completely elementary and often easy to apply, is undoubtedly well-known. However, we know of no published account of the method with the exception of an example due to Holzsager [6].

Throughout this paper all spaces are tacitly assumed to be connected, of finite type, and of the homotopy type of a CW-complex of finite type. $X_{(p)}$ denotes the localization of the space X at p and homology is taken with Z_p-coefficients for p prime unless otherwise stated. Σ^n denotes the n-fold suspension functor (reduced).

0. A general observation about self-maps

Evidently $\Sigma^n X$ splits into a wedge of the form $A_1 \vee \cdots \vee A_k$ provided $\Sigma^n X$ is equipped with self-maps which yield an orthogonal decomposition of $\tilde{H}_*\Sigma^n X$. We make this precise:

DEFINITION 0.0. $(\Sigma^n X)_{(p)}$ is said to be equipped with splitting maps $f_1, \ldots, f_k$ if the f_i are self-maps of $(\Sigma^n X)_{(p)}$ such that if $M_i = f_{i*}\tilde{H}_*\Sigma^n X$, then $f_{i*}(M_j) = 0$ for $i \neq j$, $f_{i*}(M_i) = M_i$ (and $f_{i*}\colon M_i \to M_i$ is an isomorphism), and $\tilde{H}_*\Sigma^n X \cong M_1 \oplus \cdots \oplus M_k$.

PROPOSITION 0.1. *$(\Sigma^n X)_{(p)}$ has the homotopy type of $A_1 \vee \cdots \vee A_k$ where $\tilde{H}_*A_i \cong M_i$, $i = 1, \ldots, k$, if and only if $(\Sigma^n X)_{(p)}$ is equipped with splitting maps $f_1, \ldots, f_k$.*

Received May 16, 1975.