RANGES OF HYPONORMAL OPERATORS

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It is shown that if T is a hyponormal operator on a Hilbert space H, if δ is a closed subset of the plane, and if g: $\mathbf{C}\backslash\delta \to H$ is a bounded function such that $(T - \lambda)g(\lambda) \equiv x$ for some $x \in H$, then there exists a (unique) analytic function f: $\mathbf{C}\backslash\delta \to H$ such that $(T - \lambda)f(\lambda) \equiv x$ (see Theorem 1). In case T is normal (or subnormal), the result is due to Putnam [7]; and in case T is spectral (or subspectral), the result is due to Fong and Radjabalipour [5, Lemma 2]. Actually, Putnam assumes no boundedness on g, while Fong and Radjabalipour show that the boundedness condition is necessary. (As in the case of hyponormal operators the necessity of the boundedness of g is an open question.) As an application of the above result we will show that if T is a cohyponormal operator, if S is a hyponormal operator, if W is an operator with a finite-dimensional null space, and if $WT = SW$, then T is normal (see Theorem 3). This answers a question raised by Stampfli and Wadhwa in [12, Remark to Theorem 3]; it is also a generalization of some results due to Stampfli, Wadhwa [12], Fong and Radjabalipour [5]. As byproducts we will also improve some results due to Stampfli (see Propositions 1 and 2).

From now on by an operator we mean a bounded linear transformation defined on a fixed separable Hilbert space H. The separability restriction will result in no loss of generality. The range and the null space of an operator T will be denoted by $R(T)$ and $N(T)$ respectively.

Recall that if T is normal or if the interior of the point spectrum $\sigma_p(T)$ of T is empty, then T has the single-valued extension property, i.e., there exists no nonzero, analytic, H-valued function f such that $(T - \lambda)f(\lambda) \equiv 0$. In particular every hyponormal operator has the single-valued extension property. Moreover if T has the single-valued extension property and if the manifold

$$X_T(\delta) = \{x \in H\text{: there exists an analytic function } f_x\text{: } \mathbf{C}\backslash\delta \to H \text{ such that } (T - \lambda)f_x(\lambda) \equiv x\}$$

is closed for some closed set δ, then $\sigma(T \mid X_T(\delta)) \subseteq \delta \cap \sigma(T)$ [3, Proposition 3.8, p. 23].

We first prove the following modest generalization of Theorem 2 of [11]. The result is known in case T has no residual spectrum.

PROPOSITION 1. *If T is hyponormal, then $X_T(\delta)$ is closed for all closed sets δ.*

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