THE TOTAL CHERN AND STIEFEL-WHITNEY CLASSES ARE NOT INFINITE LOOP MAPS

BY

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1. Introduction

Let Λ be a discrete commutative ring with identity. If X is a space then

$$F(X; \Lambda) = \left\{ x \in 1 + \prod_{q \geq 1} H^q(X; \Lambda) \right\}$$

has a group structure given by restricting the cohomology cup-product in $H^*(X; \Lambda)$. Also

$$F_e(X; \Lambda) = \left\{ x \in 1 + \prod_{q \geq 1} H^{2q}(X; \Lambda) \right\}$$

can be made into a group by the same device. $F(X; \Lambda)$ and $F_e(X; \Lambda)$ are representable as groups by H-space structures on

(1.1) $$K(\Lambda) = \prod_{q \geq 1} K(\Lambda; q) \quad \text{and} \quad K_e(\Lambda) = \prod_{q \geq 1} K(\Lambda; 2q)$$

respectively.

G. B. Segal has proved the following:

1.2. THEOREM [8]. *There are connected cohomology theories* $F^*(X; \Lambda)$ *and* $F_e^*(X; \Lambda)$ *such that* $F^0(X; \Lambda) = F(X; \Lambda)$ *and* $F_e^0(X; \Lambda) = F_e(X; \Lambda)$.

The total Stiefel-Whitney and Chern classes, w and c, respectively, are well-known natural homomorphisms. Details are to be found in [4, p. 229].

(1.3) $$w: \tilde{K}O(X) \to F(X; Z/2), \qquad c: \tilde{K}U(X) \to F_e(X; Z).$$

Both $\tilde{K}O(X)$ and $\tilde{K}U(X)$ extend to well-known connected cohomology theories $bo^*(X)$ and $bu^*(X)$, respectively. Details of connected K-theory may be found in [1].

In [8, Section 4] Segal asks: Does either w or c extend to a stable natural transformation between cohomology theories?

(1.4) $$w: bo^*(X) \to F^*(X; Z/2), \qquad c: bu^*(X) \to F_e^*(X; Z).$$

The representing spaces for $bo^0(\square)$, $bu^0(\square)$, $F(\square; Z/2)$, and $F_e(\square; Z)$ are infinite loopspaces. Segal's question may be equivalently rephrased: Does either w or c extend to a map of infinite loopspaces?

Received December 16, 1975.