A CHARACTERIZATION OF $PSL(4, q)$, q EVEN, $q > 4$

BY

PETER J. LAMBERT[1]

1. Introduction

We prove the following result:

THEOREM. *Let G be a group with the same character table as $PSL(4, q)$, q even, $q > 4$. The $G \simeq PSL(4, q)$.*

The argument hinges on properties of G derived from the class algebra in Section 3 which, taken in conjunction with Suzuki's work on (C)-groups in [8] and [9], enable us to obtain three subgroups of G isomorphic to $SL(2, q)$ satisfying the conditions of K. W. Phan's characterization of special linear groups in [7].

The theorem has already been proved by different methods when $q = 2$ ($PSL(4, 2) \simeq A_8$; see [4]) and when $q = 4$ (see [6]). In the present proof, all results up to and including (6.1) can be obtained for the case $q = 4$ with a little extra difficulty. But Phan's theorem in [7], which excludes the case $q = 4$, is not so easily adaptable.

The reader is referred to Section 2 of [4] where techniques of obtaining group-theoretical information from a character table are discussed: results 2.1–2.8 in [4] will be used frequently in the present paper and will be referred to henceforth as A2.1–A2.8. Most of the notation is standard; if $g \in G$ then $o(g)$ denotes the order of g and if n is a positive integer then $\pi(n)$ denotes the set of primes dividing n.

2. Products of transvections in $SL(4, q)$

A *transvection* in $SL(4, q)$ is a conjugate of

$$\begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ 1 & & & 1 \end{pmatrix}.$$

The $q - 1$ nonidentity elements in the center of the Sylow 2-subgroup

$$\left\{\begin{pmatrix} 1 & & & \\ * & 1 & & \\ * & * & 1 & \\ * & * & * & 1 \end{pmatrix}\right\}$$

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[1] The author's current address is 132 Eastholme Drive, York, England.