ON CHEN'S ITERATED INTEGRALS

BY

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Introduction

In a series of papers, Kuo Tsai Chen has introduced his "iterated integrals"; and in particular in [1] he has related them to the homology of the loop-space of a "differential space." Here, the notion of a "differential space" is very weak-C^∞-manifolds being a special case. For a differential space X there still is a deRham complex Λ^*X and a Stokes map $\rho: \Lambda^*X \to C^*X$ but one cannot, in general, assert that ρ is a homology isomorphism. The path space P_SX and the loop space Ω_SX—slightly restricted to "smooth paths"—are again differential spaces; and the "iterated integrals" can be regarded as a morphism

$$I: B^*(\Lambda^*X) \to \Lambda^*P_SX$$

where B^* is the "bar construction." Suppose now that $A^* \subset \Lambda^*X$ is a sub DGA-algebra. Then denote the image of

$$B^*(A^*) \longrightarrow B^*(\Lambda^*X) \xrightarrow{I} \Lambda^*P_SX \xrightarrow{h} \Lambda^*\Omega_SX$$

where h is the restriction, by $\int A^*$. $\int A^*$ turns out to be a sub DGA-algebra of $\Lambda^*\Omega_SX$ and "Chen's theorem" is roughly (for a precise statement see [1, 4.7.1] or 2.3 below) that if $\rho \mid A^*: A^* \to C^*X$ is a homology isomorphism, then $H^*(\int A^*) \approx H^*(\Omega X)$. Chen proves this by a pairing of $\int A^*$ with the cobar construction, using the methods of [3]. This is fairly complicated and, at least without considerable modification, restricted to simply connected spaces.

The present paper is intended to clarify the significance of the integration map I. Also, in Chapter 2, we give a simpler proof of Chen's theorem, avoiding the use of the Adams construction, and arriving at our form of the theorem, namely (roughly again): Chen's theorem is true whenever the Adams-Eilenberg-Moore theorem $H^*(\Omega X) \approx H^*(B^*(C^*X))$ is true; it is known that this is so in certain nonsimply connected cases. In some recent papers, e.g., [2], Chen has tackled these cases by a different method. The main idea of our paper is to relate iterated integrals to the category DASH of "strongly homotopy multiplicative maps," cf. [4].

We observe that, using the proof in [5], the Stokes map ρ can be extended to a map of DASH:

$$P_B: B^*(\Lambda^*X) \to B^*(C^*X).$$

Received March 1, 1976.

[1] Supported in part by a grant from the National Science Foundation.