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THE ASYMPTOTIC BEHAVIOR OF THE PRINCIPAL EIGENVALUE OF SINGULARLY PERTURBED DEGENERATE ELLIPTIC OPERATORS[1]

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1. Introduction

Let

$$Lu \equiv \sum_{i,j=1}^{n} a_{ij}(x) \frac{\partial^2 u}{\partial x_i \, \partial x_j} + \sum_{i=1}^{n} b_i(x) \frac{\partial u}{\partial x_i} \tag{1.1}$$

be an elliptic operator in a bounded domain Ω, degenerating on $\partial\Omega$ in such a way that

$$\sum_{i,j=1}^{n} a_{ij} v_i v_j = 0 \quad \text{on } \partial\Omega, \tag{1.2}$$

$$\sum_{i=1}^{n} \left(b_i - \sum_{j=1}^{n} \frac{\partial a_{ij}}{\partial x_j} \right) v_i \geq 0 \quad \text{on } \partial\Omega, \tag{1.3}$$

where $v = (v_1, \ldots, v_n)$ is the inward normal on $\partial\Omega$. In general, the Dirichlet problem for this operator has no solution since the corresponding solution of the stochastic differential system

$$d\xi_i = \sum_{j=1}^{n} \sigma_{ij}(\xi) \, dw_j + b_i(\xi) \, dt,$$

where $\frac{1}{2} \sum_{k=1}^{n} \sigma_{ik} \sigma_{kj} = a_{ij}$ does not exit from Ω in finite time (see, for example, [4] for more details).

For any $\varepsilon > 0$, consider the elliptic operator $\varepsilon\Delta + L$ and denote by λ_ε the principal eigenvalue; i.e., λ_ε is the smallest real number such that there exists a solution u_ε for the problem

$$\varepsilon \Delta u_\varepsilon + L u_\varepsilon = -\lambda_\varepsilon u_\varepsilon \text{ in } \Omega, \qquad u_\varepsilon = 0 \text{ on } \partial\Omega. \tag{1.4}$$

As is well known such an eigenvalue always exists. We are concerned in this paper with the asymptotic behavior of λ_ε as $\varepsilon \to 0$.

Several earlier papers consider the problem of estimating λ_ε when λ_ε is the principal eigenvalue for the operator

$$\varepsilon \sum_{i,j=1}^{n} a_{ij} \frac{\partial^2}{\partial x_i \, \partial x_j} + \sum_{i=1}^{n} b_i \frac{\partial}{\partial x_i}, \tag{1.5}$$

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