ILLINOIS JOURNAL OF MATHEMATICS
Volume 21, Number 4, December 1977

ON THE AUTOMORPHISM GROUP OF AN INTEGRAL GROUP RING, II

BY

GARY L. PETERSON

1. Introduction

Let G be a finite group, $Z(G)$ denote the integral group ring of G, and $NA(G)$ denote the group of normalized automorphisms of $Z(G)$. That is, $NA(G)$ denotes the group of ring automorphisms f of $Z(G)$ such that $f(g)$ has augmentation one for all $g \in G$. As remarked in [1] and [5], little generality is lost by studying normalized automorphisms over arbitrary automorphisms of $Z(G)$.

The objective of this paper is to extend the previously known list of metabelian E. R. groups. E. R. *groups* are groups G in which every element of $NA(G)$ has an elementary representation. Here, by saying that f in $NA(G)$ has an *elementary representation*, we mean that f can be written in the form $f = \tau_u \sigma$ where σ lies in the automorphism group of G, denoted by Aut (G), (actually σ extended linearly to $Z(G)$) and τ_u denotes conjugation by a unit u in $Q(G)$ (the group algebra of G over the rational field). In the notation of [5], saying that G is an E. R. group is equivalent to saying that $NA(G) = CP(G)$ Aut (G) where

$$CP(G) = \{\tau_u \mid u \text{ is a unit in } Q(G) \text{ normalizing } Z(G)\}.$$

Metabelian E. R. groups which have been obtained elsewhere include: (1) class ≤ 2 nilpotent groups from [7], and from [1], (2) groups with a cyclic normal subgroup of prime index, (3) groups with at most one nonlinear irreducible character, and (4) groups G in which $|G'| = 2$ or 3. In [6], it is shown that the symmetric groups are E. R. groups.

In Section 3, we will obtain a sufficient condition for a group which is a product of an abelian normal subgroup and an abelian subgroup to be an E. R. group. Using this result, we will show that groups containing a cyclic normal subgroup with an abelian supplement are E. R. groups, thereby generalizing (2) and, it turns out, (4). We will also see that groups G in which G/Z is metacyclic, Z the center of G, are E. R. groups. In Section 4, we will obtain some additional metabelian p-groups which are E. R. groups and consider a related problem on when the complement for Aut (G) in $NA(G)$ obtained in [5] for metabelian p-groups is contained in $CP(G)$.

Many of the results of this paper are taken from the author's Ph.D. thesis. The author would like to thank Professor J. E. Adney of Michigan State University for his guidance and encouragement.

Received May 19, 1976.