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DISJOINT SEQUENCES, COMPACTNESS, AND SEMIREFLEXIVITY IN LOCALLY CONVEX RIESZ SPACES

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1. Introduction

A number of recent results [6], [7], [11], [12] give characterizations of topological properties of Banach lattices in terms of disjoint sequences. The present work extends a number of these results to the setting of locally convex Riesz spaces. Our method is based largely upon techniques given by Fremlin [5] rather than the representation theorems of Kakutani for abstract L and M spaces, and the principal technical tools for the work are given in Proposition 2.1 and Proposition 2.2. The following notions are then characterized in terms of disjoint sequences: conditional weak sequential compactness (Proposition 2.3), weakly compact order intervals (Proposition 3.1), compact order intervals (Proposition 4.1), relative compactness (Proposition 4.3), semireflexivity (Proposition 5.4).

Our notation and terminology will be drawn from [5], [8] and results from these sources will occasionally be used without explicit reference. Throughout the paper L will denote an Archimedean Riesz space with order dual $L^{\sim}$. The band of normal integrals on L will be denoted by $L_n^{\sim}$. A locally convex Riesz space (L, T) is an Archimedean Riesz space equipped with a locally solid, locally convex Hausdorff topology T. We shall refer to a locally solid, locally convex topology T on L simply as a locally convex Riesz space topology on L. If $M \subset L^{\sim}$ is a separating ideal the locally convex Riesz space topology on L defined by the Riesz seminorms $x \mapsto |\phi|(|x|)$, $x \in L$, $\phi \in M$ will be denoted by $|\sigma|(L,M)$. A Riesz seminorm ρ on L is called a Fatou seminorm if $0 \leq x_\tau \uparrow_\tau x$ holds in L implies $\rho(x) = \sup_\tau \rho(x_\tau)$. We will use the following terminology of [5]. A locally convex Riesz space topology T on L is called (a) Fatou if T is defined by its continuous Fatou Riesz seminorms, (b) Levi if each T-bounded upwards directed system in L^+ has a least upper bound in L, (c) Lebesgue if $0 \leq x_\tau \downarrow_\tau 0$ holds in L implies $\{x_\tau\}$ is T-convergent to 0.

The well-known theorem of Nakano on completeness may now be stated in the following form (see [5], [2]):

NAKANO'S THEOREM. (a) *If L is Dedekind complete and if T is a Fatou locally convex Riesz space topology on L then each order interval of L is T-complete.*

(b) *If L is Dedekind complete and if T is a Levi Fatou locally convex Riesz space topology on L then L is T-complete.*

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