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RECURSION IN THE EXTENDED SUPERJUMP

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The history of the investigation into recursion in the superjump is a long and complicated one, with many people contributing pieces of information. The final word on the type three object defined first by R. O. Gandy [3],

$$S(F, \alpha. e) \simeq \begin{cases} 0 & \text{if } \{e\}(\alpha, F) \text{ converges} \\ 1 & \text{otherwise,} \end{cases}$$

was had by Leo Harrington [4], [5] after Peter Aczel and Peter Hinman obtained partial results. The results obtained were:

(A) The first ordinal not recursive in the superjump, ω_1^S, equals ρ_0, the first recursively Mahlo ordinal.

(B) 1-sc $S = L\rho_0 \cap 2^\omega$ where $L\rho_0$ is the collection of sets constructible before ρ_0.

The basic interest in the superjump stems from the fact that, unlike the *normal* type three objects, which involve ineluctibly uncountable computations, the superjump applied to a type two object can be viewed as a countable computation. This can be seen more clearly by replacing the superjump by the equivalent

$$\mathscr{E}(F) \simeq \begin{cases} 0 & \text{if } \exists \alpha \varepsilon \text{ 1-sc } F[F(\alpha) = 0] \\ 1 & \text{otherwise.} \end{cases}$$

Then, of course, we see that the value of $\mathscr{E}$ applied to F only depends on 1-sc F. The fact that S (and $\mathscr{E}$) are strictly weaker than ${}^3E$ makes it impossible to apply the techniques of Shoenfield and Sacks without alteration, and it is the reason that the analysis of recursion based S has been so difficult.

After result (B) above, the situation remained unsatisfactory, because of the fact that 1-env $S = \Pi_2^1$. As has been noted by Harrington, and others, this fact arises because some computations from the superjump may diverge for "the wrong reasons." For example, if a λ-term which defines a partial type two object $\dot{F}$ arises and is taken as an argument for S, the computation will diverge because $\dot{F}$ is not total, even though $\dot{F}$ may be defined at all "relevant" objects, for example, on 1-sc $\dot{F}$.

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