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SOLVABLE GROUPS ADMITTING AN "ALMOST FIXED POINT FREE" AUTOMORPHISM OF PRIME ORDER

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1. Introduction and notation

In [9], Thompson has proved that a group admitting a fixed point free automorphism of prime order is necessarily nilpotent. In this paper, we relax somewhat the fixed point free hypothesis on the automorphism, but we do assume that the group in question is solvable. The specific hypothesis considered is the following:

Hypothesis 1.1. P is a group of prime order p, N is a solvable group and P acts on N as a group of automorphisms in such a way that for every prime divisor r of $|N|$, $[R, P] = R$ holds for every P-invariant Sylow r-subgroup R of N.

If p is not a Fermat prime (i.e., p is not of the form $1 + 2^s$) then the group N in the above hypothesis is necessarily nilpotent. This fact is a consequence of results appearing in a paper of E. Shult [8], although it is not explicitly stated there. A complete proof is given here.

The interesting case, occupying the bulk of this paper, is when p is a Fermat prime. In Section 4 we show that if $p \geq 17$, then N has a nilpotent normal 2-complement, equivalently, $N/\mathbf{F}(N)$ is a 2-group, where $\mathbf{F}(N)$ is the Fitting subgroup of N. For the remaining Fermat primes (3 and 5), $N/\mathbf{F}(N)$ need not be a 2-group, but some of its structure is determined. In particular, the possible prime divisors of the order of $N/\mathbf{F}(N)$ are determined (see Theorem 4.2(c)).

Whenever one group A acts on another group B as a group of automorphisms, the usual semidirect product AB may be constructed, and this idea is used implicitly throughout this paper. One frequent occurance of this is the case when B is an $F[A]$-module for some field F. Another obviously is $A = P$ and $B = N$ in the situation of hypothesis 1.1. Notice that this hypothesis is an example of a coprime action, as $|N|$ is necessarily prime to p.

The notation used throughout this paper is standard we hope, and we use [3] and [5] as general references for the standard group theoretical results needed. We also use [2] as a general reference for representation theory.

If G is a finite group, Irr (G) denotes the set of irreducible (complex) characters of G, and for $\chi \in$ Irr (G), let det $\chi \in$ Irr (G) denote the linear character

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