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SEMI-FREE GROUP ACTIONS

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1. Introduction

In the study of equivariant differential topology, there has been considerable recent interest in the classification of semi-free actions; i.e. differential actions of a compact Lie group with every isotropy subgroup being either the entire group or the unit subgroup. This interest stems largely from the fact that free actions are "understood" and that the next simplest case is the semi-free case to which recently developed tools apply nicely.

The question with which this paper will be concerned is: Given a compact Lie group G, which unoriented bordism classes of compact manifolds contain a representative on which G acts semi-freely?

The answer is trivial, of course, for every manifold M admits a semi-free action which is trivial; i.e. every isotropy group is the entire group, and so every class contains such a representative. This trivial case must be excluded, and one seeks those classes which contain a representative M on which G acts semi-freely and non-trivially in the sense that no component of M consists entirely of points fixed by G. This problem is somewhat less trivial.

In Section 2, the necessary general nonsense of setting up appropriate bordism groups will be accomplished. In addition, the problem will be reduced to a special class of groups—those G which admit an orthogonal representation which is free on the corresponding sphere, and which are not finite of odd order. In Section 3, Conner and Floyd's methods will be used to compute the bordism groups partially, or rather theoretically. This reduces the problem to understanding the classifying space for principal G bundles and the representations of G. In addition the case $G = Z_2$ will be thoroughly studied, since everything will map into this case.

In Section 4, the work really begins. Based on the general nonsense, and the partial calculations, one can state exactly what will be proved. This reduces the problem to verifying certain properties for each group involved, and in essence gives the plan for the remaining sections.

In Section 5, the infinite groups admitting the appropriate representations will be studied, and in Section 6 the finite groups will be studied. In Section 7, some examples of groups will be given to show that all potential images in $\mathfrak{N}_*$ actually arise, and examples of manifolds to show that the images are distinct.

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