ILLINOIS JOURNAL OF MATHEMATICS
Volume 23, Number 4, December 1979

UNITS OF IRREGULAR CYCLOTOMIC FIELDS

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In an interesting series of papers, P. Dénes proved a number of results on cyclotomic fields, especially concerning their units, under the unproved assumption that the so called p-character of the Bernoulli numbers is finite. By relating this p-character to p-adic L-functions, we prove its finiteness as a consequence of the nonvanishing of the p-adic regulator. We then show how the p-adic regulator and p-adic L-functions may be used to obtain simple proofs of some of Dénes' results. We also show that a formula of Dénes is essentially the same as Leopoldt's p-adic class number formula. Finally, we give an application to the second case of Fermat's Last Theorem.

Since the p-adic L-functions are essentially an embodiment of many of the classical congruences for Bernoulli numbers (e.g., Kummer's congruences), several of our proofs can probably be translated back to the original ones, which relied heavily on properties of Kummer's logarithmic differential quotient. But the use of the theory of p-adic L-functions seems to be much more natural and also leads to new interpretations of classical results, in addition to being essential to the proof that the p-character is finite.

1. The p-character of the Bernoulli numbers

Throughout this paper we shall assume $p \geq 5$. Let the Bernoulli numbers be defined by

$$\frac{te^t}{e^t-1} = \sum_{n=0}^{\infty} B_n \frac{t^n}{n!},$$

so $B_0 = 1$, $B_1 = \frac{1}{2}$, $B_2 = \frac{1}{6}$, $B_3 = 0$, etc. (Dénes [2]) defined the p-character of the Bernoulli numbers to be the integers $u_2, u_4, \ldots, u_{p-3}$ defined by

$$B_{ip^j} \equiv 0 \pmod{p^{2j+1}} \quad \text{for} \quad 0 \leq j < u_i,$$
$$B_{ip^{u_i}} \not\equiv 0 \pmod{p^{2u_i+1}},$$

where $i = 2, 4, \ldots, p-3$. If p is a regular prime, then $u_i = 0$ for all i. We shall show how these numbers relate to p-adic L-functions.

Let $\mathbf{Z}_p$ be the ring of p-adic integers. If $a \in \mathbf{Z}_p$ and $p \nmid a$, then there is a unique $(p-1)$st root of unity $\omega(a) \in \mathbf{Z}_p$ such that $\omega(a) \equiv a \pmod{p}$. We may regard ω as a p-adic valued Dirichlet character. Define $\langle a \rangle = a/\omega(a)$, so

Received November 21, 1977.

[1] Research supported in part by a National Science Foundation grant.