ILLINOIS JOURNAL OF MATHEMATICS
Volume 24, Number 1, Spring 1980

LINK COMPLEMENTS AND COHERENT GROUP RINGS

BY

PHILIP S. HIRSCHHORN

1. Introduction

A link of multiplicity m is an embedding of the disjoint union of m copies of the $(q-2)$-sphere S^{q-2} in the q-sphere S^q. (We will work in the smooth category, but all statements will be true when said for the piecewise linear category as well.) Any attempt to classify links analyzes the complement of the embedded spheres; thus, one would like to have a characterization of the homotopy type of a link complement.

By Alexander duality, a link complement C always has the same homology groups as the wedge $(\vee^m S^1) \vee (\vee^{m-1} S^{q-1})$. Gutierrez [4] has shown that for $q \geq 6$, if $\pi_i C \cong \pi_i(\vee S^1)$ for $i \leq (q-1)/2$ and if $\pi_1 C$ is generated by the meridians, then the link is trivial. We will consider those links for which there is an integer n satisfying $1 \leq n \leq (q-3)/2$, such that $\pi_i C \cong \pi_i(\vee S^1)$ for $i \leq n$. Thus, all fundamental groups will be isomorphic to $F(m)$, the free group on m generators, and all higher homotopy groups will be modules over $\Phi = \mathbf{Z}[F(m)]$, the group ring of the free group on m generators.

If G is a group and M is a $\mathbf{Z}[G]$-module, the *homology groups* $H_*(G; M)$ are defined as the homology groups of a $K(G, 1)$ with (twisted) coefficients in M, and a $\mathbf{Z}[G]$-module M is *acyclic* if $H_i(G; M) \cong 0$ for all $i \geq 0$.

Our main results are then the following two theorems.

THEOREM 1. *Let n and q be integers satisfying $1 \leq n \leq (q-3)/2$, and let C be the complement of a link $f\colon \cup S^{q-2} \to S^q$ such that $\pi_i C \cong \pi_i(\vee S^1)$ for $i \leq n$. Then $\pi_i C$ is a finitely presented acyclic Φ-module for $n+1 \leq i \leq 2n$.*

THEOREM 2. *Let n be a positive integer, and let X be a space for which $\pi_i X \cong \pi_i(\vee S^1)$ for $i \leq n$, $\pi_i X$ is a finitely presented acyclic Φ-module for $n+1 \leq i \leq 2n$, and $\pi_i X \cong 0$ for $i \geq 2n+1$. Then for any $q \geq 4n+3$ there is an embedding $f\colon \cup S^{q-2} \to S^q$ with complement C, and a map $C \to X$ inducing isomorphisms $\pi_i C \cong \pi_i X$ for $i \leq 2n$.*

A key technical point is that Φ is a *coherent* ring, which we discuss in Section 2, along with some properties of acyclic Φ-modules. Theorems 1 and 2 are proved in Sections 3 and 4, respectively.

Received March 14, 1978.