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A STRONG SPECTRAL RESIDUUM FOR EVERY CLOSED OPERATOR

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1. Introduction

Decomposable operators (see, e.g., [2]) are linear operators, for which a weaker, geometric variant of the constructions, characteristic of spectral operators [3], is still possible. Residually decomposable operators, introduced by F.-H. Vasilescu [6], [7], and bounded S-decomposable operators, studied by I. Bacalu [1], are operators such that, loosely speaking, the property of decomposability holds only outside a certain part of the spectrum. F.-H. Vasilescu has proved [7] that for certain operators having the single-valued extension property there is a unique minimal closed subset of the spectrum, called the spectral residuum, outside which the operator has a good spectral behavior of this kind.

The main result of this paper is that, utilizing a similar concept of good spectral behavior, for an arbitrary closed operator there exists a unique minimal closed subset of the spectrum, called the strong spectral residuum, outside which the operator shows this behavior. It is proved that for a large class, close to that occurring in [7; Theorem 3.1], of operators strong and ordinary spectral residues coincide. If the strong spectral residuum is void, the operator is (bounded and) decomposable. Whether the converse is true, is equivalent to a well-known unsolved problem, raised by I. Colojoară and C. Foiaş [2; 6.5 (b)]. Though the proofs seem to remain valid after minor modifications in a Fréchet space, to make references more convenient, we have chosen the Banach space setting.

Let X be a complex Banach space and let $C(X)$ and $B(X)$ denote the class of closed and bounded linear operators on X, respectively. Let C and $\bar{C}$ denote the complex plane and its one-point compactification, respectively. Unless stated explicitly otherwise, all topological concepts for sets in $\bar{C}$ will be understood in the topology of $\bar{C}$. If $F \subset \bar{C}$, then F^c denotes $\bar{C} \backslash F$ and $\bar{F}$ denotes the closure of F. For $T \in C(X)$, $D(T)$ is its domain and $\sigma(T)$ denotes its extended spectrum, which coincides with the spectrum $s(T)$ if $T \in B(X)$, and is $s(T) \cup \{\infty\}$ otherwise. We set $\rho(T) = \sigma(T)^c$. If Y is a closed subspace of X and $T(Y \cap D(T)) \subset Y$, then we write $Y \in I(T)$ and $T \mid Y$ denotes the restriction of T to $Y \cap D(T)$.

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