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ON THE FOURIER SERIES OF CERTAIN SMOOTH FUNCTIONS[1]

BY

CALIXTO P. CALDERÓN AND YORAM SAGHER

1. Introduction and statement of results

By $w(t) = w(f, t)$ we shall denote the L^1-modulus of continuity of a period function belonging to $L^1(-\pi, \pi)$, namely

$$w(t) = \sup_{|h|<t} \int_{\pi}^{\pi} |f(x+h) - f(x)|\, dx. \tag{1.1}$$

A classical result of Marcinkiewicz shows that if

$$\int_0^1 w(t)\frac{dt}{t} < \infty,$$

then the Fourier Series of f converges a.e. The aim of this paper is to show a connection between the smoothness of a function and the growth of the partial sums of its Fourier Series.

THEOREM 1. *Suppose that $w(f, t) < c/|\log t|$; then*

$$S_n(f) = o[\log \log n(\log \log \log n)^{1+\varepsilon}] \quad \text{a.e.} \quad \varepsilon > 0.$$

More generally:

THEOREM 2. *Let $w(t)$ be the L^1-modulus of continuity of f. Let $\phi(t)$ be a continuous increasing function of the variable t such that*

$$\int_0^1 w(t)\phi(t)\frac{dt}{t} < \infty, \quad \phi(0) = 0.$$

Then

$$S_n(f) = o\left(\phi\left[\frac{1}{n}\right]\right)^{-1} \text{ a.e.}$$

REMARK. If $w(t)$ satisfies the Dini condition, there $S_n(f)(x)$ converges a.e. On the other hand, the closer $w(t)$ gets to satisfying the Dini condition the slower the growth of $S_n(f)$ is.

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