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ℵ-PROJECTIVE SPACES

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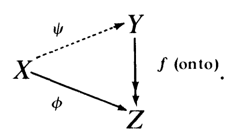
1. Introduction

By a space we shall always mean a compact Hausdorff space, a map shall always be a continuous map between spaces, and a diagram shall always be a commutative diagram of spaces and maps. A space X is projective if the following lifting property holds. Given spaces Y and Z and maps $\phi: X \to Z$ and $f: Y \to Z$ with f onto, there exists a map $\psi: X \to Y$ satisfying $\phi = f \circ \psi$. In other words, a solution ψ exists in any diagram

(1)



We call ψ a lifting of ϕ over f. A well known theorem of Gleason characterizes the projective spaces as the extremally disconnected spaces [5][2, p. 51]. A space is extremally disconnected if open sets have open closures.

The weight wt (X) of a space X is the least cardinal of a base of open sets. Let $\aleph$ be an infinite cardinal. We shall say that a space X is $\aleph$-projective if a solution ψ exists in diagram (1) whenever the additional condition wt $(Y) < \aleph$ is satisfied. Since f is onto, wt $(Z) < \aleph$ is also implied; but note that wt (X) is not mentioned. The purpose of this paper is to give the following characterization of $\aleph$-projective spaces.

THEOREM 1. *For $\aleph > \aleph_0$, a compact Hausdorff space X is $\aleph$-projective iff it is a totally disconnected $F_\aleph$-space.*

The following definitions are more or less standard; we follow the conventions of [2]. A cozero set in a space is the complement of the set of zeros of a continuous real valued function, and a set is $\aleph$-open if it is the union of fewer than $\aleph$ cozero sets. A space is an $F_\aleph$-space if any two disjoint $\aleph$-open sets have disjoint closures. An $F_{\aleph_0}$-space is called an F-space. An $\aleph_1$-open set is a cozero set, so an F-space is also an $F_{\aleph_1}$-space. Any space X is $\aleph_0$-projective, and we shall ignore this trivial case from now on.

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