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$K(Z, 0)$ AND $K(Z_2, 0)$ AS THOM SPECTRA

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Let X be a connected space. For a map $f\colon X \to BO$ or $f\colon X \to BF$, one can choose a filtration of f by restrictions $F_pX \to BO(n_p)$ or $F_pX \to BF(n_p)$, pull back the universal bundles or spherical fibrations, take Thom complexes, and so obtain a Thom spectrum Mf. These Thom spectra of maps were first introduced and studied by Barratt [unpublished] and Mahowald [10], [11]; their original work in this direction dates back to the early 1970's. A detailed analysis of this construction has recently been given by Lewis [2], [8]. Mf is independent of the choice of filtration and depends only on the homotopy class of f. For $f\colon X \to BO$, Mf is the same as $M(jf)$, $j\colon BO \to BF$. There is a Thom isomorphism under the evident orientability assumption.

If X is an H-space and f is an H-map, then Mf admits a product $Mf \wedge Mf \to Mf$ with two-sided unit (in the stable category). Here subtleties begin to enter. If X is homotopy associative, it need not follow that Mf is associative unless X and BO or BF admit associating homotopies compatible under f. However, the homology Thom isomorphism commutes with products and so the relevant homology algebras will be associative even if Mf is not. Lewis has determined the precise higher multiplicative structure present on Mf when X is an n-fold loop space and f is an n-fold loop map, but we shall not need anything so elaborate.

Mf is (-1)-connected, and $\pi_0 Mf$ is a cyclic group since Mf can be constructed to have a single zero cell. If f is non-orientable, so that $f^*(w_1) \neq 0$, then the zero cell extends over the Moore spectrum and $\pi_0 Mf = Z_2$. If f is both non-orientable and an H-map, then $\pi_* Mf$ is a Z_2 vector space and thus Mf is 2-local.

Our purpose in this note is to give a simple proof of the following striking theorem of Mark Mahowald [11].

THEOREM 1. *There is a map $f\colon \Omega^2S^3\langle 3\rangle \to BSF$ whose associated Thom spectrum is the Eilenberg-Mac Lane spectrum $K(Z, 0)$.*

Here $S^3\langle 3\rangle$ is the 3-connective cover of S^3. This is analogous to the following earlier result of Mahowald [10, 4.5].

THEOREM 2. *The Thom spectrum associated to the second loop map $\bar{\eta}\colon \Omega^2S^3 \to BO$ determined by the non-trivial map $\eta\colon S^1 \to BO$ is the Eilenberg-Mac Lane spectrum $K(Z_2, 0)$.*

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