THE TRANSFER IN SEGAL'S COHOMOLOGY

BY

A. KOZLOWSKI

1. Introduction

Let R be a commutative ring and let $G^*(X; R)$ denote Segal's cohomology with coefficients in the graded ring $R[x]$ of polynomials in one indeterminate x, of degree one if char $R = 2$ and two otherwise (see [9]).

Recall [3] that every generalized cohomology theory admits a transfer homomorphism for finite coverings such that stable transformations of cohomology theories commute with the transfer. The main purpose of this paper is to compute the transfer for the functor $G^0(X; R)$, where $R = Z/hp$, for coverings of the form

$$X \approx E(Z/p) \times X \to B(Z/p) \times X.$$

As an application of this computation we show that multiplicative operations (in the sense of Atiyah-Hirzebruch [1]) of classical cohomology with coefficients in Z/p—and in particular the total Steenrod p-th power operation (Steenrod square for $p = 2$)—when restricted to units do not extend to operations in Segal's cohomology. This result is surprising, and should be compared with the situation in K-theory localised at a prime q. In this case the Adams operations ψ^k, where k is not divisible by q, are automorphisms of connective K-theory, and their restrictions to multiplicative units extend to operations in the multiplicative (tensor product) cohomology theory described by Segal in [8] (see [6]).

Another application of the computation of the transfer in Segal's cohomology will appear in [4].

The plan of the paper is as follows. In §2 the definition and the main properties of the Kahn-Priddy transfer are recalled. In §3 the transfer in Segal's cohomology is computed. In §4 the impossibility of extending of Atiyah-Hirzebruch operations to transformations of Segal's cohomology is proved. In §5 some implications of this result and some related questions are discussed.

Finally, I wish to thank the Japan Society for the Promotion of Science for financial support under their post-doctoral fellowship scheme.