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FINITE GROUPS THAT ACT ON SPHERES IN WHICH A CENTRAL ELEMENT ACTS FREELY

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Introduction

It is known that if a finite group, G, acts freely on a homotopy $(2N-1)$-sphere then $H^*(G; Z)$ has period $2N$. In this paper we show that if G is a finite group with a central element T of order p (p a prime) and if G acts on a homotopy $(2N-1)$-sphere in such a way that T acts freely then this puts certain restrictions on the Hochschild–Serre spectral sequence for computing $H^*(G; Z_p)$, and in particular we obtain an element

$$\xi \in H^{2p^{\mu(N)}}(G; Z_p) \quad \text{where } \mu(N) = \max\{i: p^i \mid N\},$$

such that ξ is a non-zero divisor in the ring $H^*(G; Z_p)$. We can use this to prove that $H^*(G; Z_p)$ has period $2p^{\mu(N)}$ in the case that G acts freely.

In Section 1 we establish some relevant homological algebra. In Section 2 we describe a splitting lemma: namely if K acts on a homotopy lense space L then $H^*(EK \times_K L; Z_p)$ is a $H^*(K; Z_p)$ direct summand of $H^*(G; Z_p)$ where G and K are related by an extension $1 \to Z_p \to G \to K \to 1$. In Section 3 we prove the main theorems of the paper and discuss the example of extra-special 2-groups acting on a homotopy sphere in which the central element of order 2 acts freely.

1. Homological algebra

Let K be a field and Λ a finitely generated augmented K-algebra, assumed to be commutative. Let C_* always denote a Λ-chain complex such that each C_n is a finitely generated free Λ-module. Let $H_* C_*$ be the usual homology groups of C_*.

We will let $C_*^{[N]}$ denote the free Λ-chain complex constructed by killing off the cycles of C_* in dimensions $N+1$ and larger [1]. Then $C_*^{[N]}$ comes equipped with a chain map

$$j: C_* \longrightarrow C_*^{[N]}$$

and satisfies the following properties.

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