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HARMONIC AND SUPERHARMONIC FUNCTIONS ON COMPACT SETS

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In [4], T.W. Gamelin gives necessary and sufficient conditions which ensure that every continuous function on a compact subset K of $\mathbf{R}^2$, harmonic on the interior of K, can be approximated uniformly on K by functions harmonic in a neighborhood of K. Here we shall see that using [1] and [3] a stronger version of the same result can be proved even for arbitrary harmonic spaces.

In the following let K be a compact subset of a $\mathscr{P}$-harmonic space $(X, {}^*\mathscr{H})$ and let $\mathscr{C}(K)$ denote the space of all continuous real functions on K. For every finely open set V contained in K let $H(K, V)$ (resp. $S(K, V)$) be the set of all functions $g \in \mathscr{C}(K)$ such that $\varepsilon_x^{\mathbf{C}G}(g) = g(x)$ (resp. $\varepsilon_x^{\mathbf{C}G}(g) \leq g(x)$) for every $x \in V$ and every fine neighborhood G of x such that $\bar{G} \subset V$. The functions in $H(K, V)$ (resp. $S(K, V)$) are called finely harmonic (resp. finely superharmonic) on V. Evidently, $H(K, V) = S(K, V) \cap (-S(K, V))$.

This definition is useful in our context because of the following two facts. If V is open then $H(K, V)$ (resp. $S(K, V)$) is the set of all functions in $\mathscr{C}(K)$ which are harmonic (resp. superharmonic) on V [3, p. 264]. Furthermore, if V is the fine interior of K then $H(K, V)$ (resp. $S(K, V)$) is the uniform closure of the set $H(K)$ (resp. $S(K)$) of all functions in $\mathscr{C}(K)$ which are restrictions of harmonic (resp. superharmonic) functions on a neighborhood of K [1, p. 105], [3, p. 269].

A characterization of the Choquet boundary $Ch_{S(K,V)}K$ of K with respect to $S(K, V)$ involves the essential base of $\mathbf{C}V$. Let us recall that for every subset A of X the base $b(A)$ of A is the set of all points $x \in X$ such that A is not thin at x whereas the essential base $\beta(A)$ of A (called quasi-base $\rho(A)$ in [5]) is the set of all points $x \in X$ such that A is not semi-polar at x, i.e., such that for every fine neighborhood V of x the set $A \cap V$ is not semi-polar. We note that $\beta(A)$ is the smallest finely closed subset F of X such that $A \setminus F$ is semi-polar. Moreover, if A is finely closed then $\beta(A)$ is the largest subset F of A such that $b(F) = F$.

If $V \subset K$ is finely open then

$$Ch_{S(K,V)}K = K \cap \beta(\mathbf{C}V)$$

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