ILLINOIS JOURNAL OF MATHEMATICS
Volume 29, Number 1, Spring 1985

CONVEXITY AND CYLINDRICAL TWO-PIECE PROPERTIES

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Let f: $M \to \mathbf{R}^n$ be a smooth immersion of a compact manifold. In particular we say that f is trivial if M is diffeomorphic to $\mathbf{S}^{n-1}$ and f embeds M as a round hypersphere.

The idea of k-cylindrical tautness and the related k-cylindrical weak and strong two-piece properties were discussed in [2]. It was shown that the weak $(n-2)$-cylindrical two-piece property is sufficient to imply that f is trivial. It was also shown that the weak 1-cylindrical two-piece property implies that f embeds $\mathbf{S}^{n-1}$ as a tight hypersphere and the comment was made that if f is 1-cylindrically taut then it is trivial. This fact is proved here.

We also consider the case $k = 2$. We show that the weak and strong versions of the two-piece property are distinct by giving an embedding of $\mathbf{S}^1 \times \mathbf{S}^{n-2}$ in $\mathbf{R}^n$ which has the weak 2-cylindrical two-piece property and by showing that if f has the strong version and $\dim M = n - 1$ then f embeds $\mathbf{S}^{n-1}$ as a tight hypersphere. We also prove that f is trivial if it is 2-cylindrically taut and $\dim M = n - 1$. There remains the possibility of nontrivial 2-cylindrically taut immersions of codimension 2 which must have very restrictive curvature properties.

To prove these results we need some theorems about convex sets which seem of interest in themselves.

1. Preliminary notations and results

Throughout this paper M will be a smooth, compact, connected m-dimensional manifold without boundary and f: $M \to \mathbf{R}^n$ will be a smooth immersion into n-dimensional Euclidean space. If $\Pi \subset \mathbf{R}^n$ is a k-plane, not necessarily through the origin, we define the solid k-cylinder with axis the k-plane Π and radius $r > 0$ to be the set $C = \{x \in \mathbf{R}^n\colon d(x, \Pi) \leq r\}$ where $d(x, \Pi)$ is the Euclidean distance from x to Π. We write $\tilde{C}$ for the closure of $\mathbf{R}^n \setminus C$. Let us repeat for reference the definitions given in [2].

DEFINITION 1.1. The immersion f: $M \to \mathbf{R}^n$ is *k-cylindrically taut* if there exists some field $\mathbf{F}$ such that, for all solid k-cylinders C with axis Π, inclusion

Received September 14, 1982.