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CYCLIC INNER FUNCTIONS IN THE BERGMAN SPACES AND WEAK OUTER FUNCTIONS IN H^p, $0 < p < 1$

BY

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Let X denote a topological vector space of analytic functions on the unit disk so that $H^\infty \subset X$ and convergence in X implies uniform convergence on compact sets. If $f \in X$ then $[f]$ denotes the closure of $\{Pf: P \text{ is a polynomial}\}$; i.e., $[f]$ is the smallest invariant (under multiplication by z) closed subspace containing f. We say f is *X-cyclic* if $[f] = X$. We shall be concerned with the case when the function is an inner function. If q is an inner function we say that q is *X-inner* if whenever q_0 is an inner function and $q_0 \in [q]$, then q divides q_0. Initially, we shall consider a general class of Banach spaces which includes the Bergman spaces. Any of these spaces will be denoted by B. In Section 1 conditions on B are obtained so that if q is an inner function, then $q = q_1 q_2$ where q_1 is B-cyclic and q_2 is B-inner. In Section 2, with further conditions imposed on B (the Bergman spaces still satisfy these conditions), we characterize the B-cyclic and B-inner functions. In Section 3 the case when $X = H^p$, $0 < p < 1$, with the weak topology is considered. In this setting X-cyclic inner functions are called *weak outer functions* and X-inner functions are called *weak inner functions.* Using the results from Section 2 we characterize the weak inner and weak outer functions in H^p, $0 < p < 1$. Also it is shown that for a large class of singular inner functions S_μ, the quotient spaces $H^p/S_\mu H^p$ contain compact convex sets with no extreme points.

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